UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Basic Calculus - 2

2425-1-E1803M047-T2

Learning objectives

The course aims to provide students with a solid mathematical foundation essential for successfully tackling advanced courses in economics, finance, econometrics, and other quantitative disciplines. By the end of the course, students should have understood the concept of real functions of one or two variables and be able to study their fundamental properties (domain of existence, sign, symmetries, limits, asymptotes, monotonicity, concavity, differentiability, etc.). Analyzing and interpreting these functions and their properties is crucial for understanding the mathematical models widely used in economics and finance.

Some examples of applications include models for interest rates, for the pricing of stocks and derivatives in a financial market; the use of utility functions to describe an agent's preferences; the use of functions of one or more variables to describe an enterprise's cost and production functions and the related optimization problems. Additionally, functions can be used to describe the probability of future events occurring, which is essential for managing financial and non-financial risks.

More generally, the course aims to enhance students' critical and analytical thinking skills through the resolution of mathematical problems.

Contents

Study of real functions of a real variable: domain, sign, intersections with the axes, limits of functions, extremum points, monotonicity of a function, concavity, inflection points, continuity, and differentiability of a function. Introduction to sequences and functions of two variables.

Detailed program

UNIT 1 - Real functions of a real variable:
Sets N, Z, Q, R. Upper / lower bounded set; intervals; infimum / supremum / maximum / minimum extremum of a set.
Definition of functions and sequences; field of existence; definition of image, image set, counter-image, counter-image set, graph; use of the analytical expression of a function a a sequence. Use of the graph of a function; injective, surjective bijective function; lower / upper bounded functions; infimum / supremum of a function; minimum / maximum, point of minimum / maximum of a function; even / odd function; monotonicity of a function and of a sequence. Operations with functions, composition, inversion. Simple transformations of graphs. Horizontal / vertical translations, horizontal / vertical reflections; partial reflections horizontal / vertical; rescaling. Composed transformations of graphs.

UNIT 2 - Limits:
Extended real line and neighborhood; definition of internal, external, frontier, isolated, accumulation point; definition of limit for functions and sequences, right / left limit, limit from above/below; reading limits from the graph. Uniqueness of the limit theorem (with proof), theorem of permanence of the sign (with proof), squeeze theorem (with proof). Limit computation. Continuity. Algebra in R*, determined forms, limits of exponential, logarithmic functions, arctangent. Indeterminate forms, techniques for solving some indeterminate forms (functions rational / irrational). Asymptotic equivalence and properties. Infinity orders, infinity hierarchies. Negligible function (little-o). Fundamental limits and relative asymptotic equivalences. Indeterminate forms of exponential type and techniques of solution. Orders of infinitesimal, hierarchy of infinitesimal, little-o. Continuity (from right / left) and discontinuity. Classification of discontinuities. Indentification of discontinuities from the graph. Horizontal, vertical, oblique asymptotes. Weierstrass theorem with counterexamples, intermediate value theorem with counterexamples, zeros theorem with counterexamples.

UNIT 3 - Derivatives:
Newton difference quotien of a function at a point; derivative function; derivatives of elementary functions; computation of derivatives. Tangent line equation; continuity-differentiability link, vertical tangent inflection point, cusp, kink. de L'Hopital's rule; Rolle's theorem (with proof) and counterexamples; Lagrange's theorem (with proof) and counterexamples; derivative of the inverse function. Monotonicity test (with proof) and counterexamples; definition of local extrema; stationary point; Fermat's theorem (with proof); definition of critical point; first derivative test for internal extrema. Monotonicity study for sequences. Subsequent derivatives test; first derivative test for boundary extrema; concave / convex function definition; first order test for concavity; second order order for concavity; inflection point. Taylor and McLaurin polynomials; use of Taylor polynomial for limit computation.

UNIT 4- Complete function study and two variables-functions:
General scheme for the study of a function. Analytical and graphical domains for real functions of two real variables; level curves; partial derivatives, gradient, stationary points

Prerequisites

Set theory. Powers, logarithms, exponentials and their properties.
First and second degree inequalities, rational inequalities, logarithmic and exponential inequalities. Cartesian equations of the line, of the circumference, of the parabola, equation of the line passing through two points. Elements of trigonometry.

Teaching methods

A hybrid teaching approach is used, combining lecture-based (DE) and interactive teaching (DI) methods. The DE includes the presentation and detailed explanation of theoretical content, typically occurring in the first part of the lesson. The DI involves active student participation through answering questions and problems posed by the instructor, short presentations, and group discussions, usually conducted in the second part of the lesson. The exact number of hours dedicated to DE and DI cannot be predetermined, as the methods intertwine dynamically to adapt to the course needs, fostering participatory and integrated learning by combining theory and practice.

Specifically:

36 hours will be conducted in person with the hybrid teaching method described above.
4 hours will be delivered remotely (asynchronously) through short videos summarizing the key concepts needed for the exam.
12 hours of exercises will be conducted in person in an interactive manner.

Assessment methods

Assessment Methods
The written exam lasts two hours and consists of 5 exercises and 3 open theoretical questions.
The open questions evaluate the student's ability to use correct mathematical language, understand the theorems and proofs discussed in class, and apply the logical steps used in these proofs.

The structure of the exercises is as follows:

Exercise 1: Transformations of Graphs of Elementary Functions
This exercise assesses the student's ability to draw the graph of a function based on the graphs of elementary functions.

Exercise 2: Limits
This exercise evaluates the student's ability to determine the best technique for solving limits.

Exercise 3: Various
This exercise assesses the student's ability to apply theorems or methodologies studied in the course that are not strictly related to the study of functions.

Exercise 4: Functions of Two Variables
This exercise evaluates the student's ability to work with functions of multiple variables.

Exercise 5: Complete Study of a Function
This exercise assesses the student's ability to analyze and understand the properties of a function.

The written exam evaluates the formal correctness of the steps, the appropriateness of the mathematical language used, and the skills and knowledge acquired during the course.

After passing the written exam, either the professor or the student may request an additional oral exam. The oral exam covers the entire course content and can positively or negatively affect the final grade.

The course does not include splitting the exam into intermediate tests.

Textbooks and Reading Materials

Textbooks

Guerraggio, A. Matematica 4/Ed. • con MyLab. Pearson.

Additional textbooks

Torriero, A., Scovenna M., Scaglianti, L.: Manuale di matematica. Metodi e applicazioni. CEDAM
Scovenna, M., Grassi, R.: Matematica – Esercizi e temi d'esame. CEDAM.
Monti, G., Pini, R.: Lezioni di matematica generale: funzioni reali di variabile reale, L.E.D.

Further learning material
Lecture notes (available on the e-learning platform).
Texts and solutions of past written exams (available on the e-learning platform).
List of demonstrations that can be requested and examples of theory questions (available on the e-learning platform).

Semester

First term, first year

Teaching language

Italian

Sustainable Development Goals