COURSE SYLLABUS

Linear Algebra

2425-1-E4102B002

Learning objectives

This course aims to provide the foundations of linear algebra that will form the basis of the courses of Probability and Multivariate Statistical Analysis.

Knowledge and understanding

This course will provide knowledge and understanding in relation to:

- Representation of vector spaces and systems of generators and bases
- Linear applications and their relationship with matrices and linear systems
- Orthogonal projections
- Role of eigenvalues and eigenvectors of a matrix

Ability to apply knowledge and understanding

At the end of the course the students will be able to:

- Study linear applications through the theory of matrices and linear systems
- Determine the best approximation of an element of a vector space among the elements of a subspace
- Apply the procedure of diagonalization of matrices

The course allows the student to acquire a solid foundation in the use of linear algebra necessary in any work context and representing an essential basis for the continuation of the university studies.

Contents

- Systems of linear equations and matrices

- Vector spaces and linear maps
- determinant
- similarity of matrices and diagonalization
- orthogonal projections
- congruence of matrices and quadratic forms

Detailed program

1. Systems of linear equations and their solution
2. Algebra of matrices. Product between matrices. Invertible matrices. Rank of a matrix.
3. Vector spaces on R. Linear combinations and independence. Systems of generators and bases. Dimension of a vector space.
4. Linear maps. Study of linear maps by matrices and linear systems. Rank plus nullity Theorem.
5. Determinant and its computation.
6. Eigenvalues and eigenvectors. Diagonalization procedure.
7. Bilinear forms and inner products. Orthonormal bases. Orthogonal projections.
8. Spectral Theorem and congruence of matrices
9. Quadratic forms.
10. Applications to statistics

Prerequisites

No prerequisites are required.

Teaching methods

48 hours of in-person, lecture-based teaching (6 ECTS). Classic frontal lessons, partly devoted to the theoretical aspects of the course, and partly to the resolution of practical exercises, which allow the student to acquire a method and a logical approach in solving problems. To practice problems at home and during exams we will make use of a online platform of exercises.

Assessment methods

The exam consists of a computerized written test plus a possible oral exam. The exam consists of exercises similar to those seen in the online platform of exercises accissible from home, and questions with more theoretical content. The test is computerized, but the answer to some types of questions / exercises can also be requested on paper. The duration of the test is 2 hours and during the use of textbooks or notes and handouts is not allowed. The use of programmable calculators is not allowed.

The oral part of the exam is **not** mandatory for everyone.
-Students whose grade for the written exam is 16 or less will have to take the exam again at a later time.
-The oral part of the exam is *mandatory* for everyone whose grade for the written exam is either 17, 18 or 19.
-The oral part of the exam is *elective* for those attaining a grade is greater or equal to 27 in the written exam: such

students can choose to either accept a final grade of 27 or take the oral part of the exam to achieve an even better grade (it being understood that if the oral part of the exam is unsatisfactory it can lead to a worsening of the grade and even to a failure of the whole exam). To be clear: a student achieving a 28 in the written part of the exam can choose to register a final grade of 27 (and not take the oral part of the exam) or to take the oral part of the exam: depending on the oral part of the exam the 28 can either be lowered (even to a possible failure of the exam), confirmed or improved.

- To those whose grade for the written part of the test is at least 17: the oral part of the exam must be taken in all cases that either the instructor or the student requests it be taken (if a student wants to improve the grade of the written part of the exam). For example, a student who got 24 in the written test might want to skip the oral part, but also take it to improve the grade.
- The oral part of the exam, when mandated, needs to be taken in the same round (*appello*) of the written part of the exam. In all cases, the exam ends in the same round of a written part.

During the course of the lectures students will be allowed to practice solving problems on the computerized system from their homes and, in case they work on all the problems during the required time frame, they can gain up to 2 extra points. The bonus points will be added to the grade of the written exam, allowing them to have an improved grade for the written part of the exam.

Textbooks and Reading Materials

1. Schlesinger E., Algebra Lineare e Geometria, Zanichelli, 2017 (second edition)
2. Fioresi R.,Morigi M., Introduzione all'Algebra Lineare, Casa Editrice Ambrosiana, 2021 (second edition)
3. Notes of Lecures available on this platform.

Semester

II semester, III and IV cycle (approximately from March to June).

Teaching language

Italian.

Sustainable Development Goals

QUALITY EDUCATION