UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Derivatives

2425-1-F1601M051-F1601M056M

Learning objectives

1) Know the basic types of derivative instruments and understand their uses and financial significance

2) Understand the concept of a mathematical model of a financial market and its use in the valuation of a derivative instrument

3) To know in detail the models studied and the derivations of their formulae

4) Be able to apply the studied models to the valuation and hedging of a generic derivative instrument

5) Know how to access the Bloomberg terminal and understand the meaning of the main functions related to equity derivatives: OMON, OVME, SKEW.

Contents

- Basic concepts on options

- Multi-period binomial model

- One-period models

- Continuous-time models

- Black-Scholes model

- Merton model

- The Monte Carlo method and delta-hedging

- The VIX Index

- The main Bloomberg functions on options: OMON, OV, OSA, SKEW.

Detailed program

Basic Concepts on Options
Recalls on derivative instruments: forward contracts, futures contracts, call and put options and examples of applications. Payoff and replication concept; forward price and spot-forward parity. Put-call parity. Combinations of options (spread, butterfly, strangle, straddle). Convexity of the call price as a function of the strike. Super-replication and sub-replication. Merton constraints. American options and optimality of early exercise. Qualitative discussion of factors influencing option prices.

Multi-period binomial model
The one-period binomial model: derivation of the formula for the price of a generic payoff.
The biperiodal binomial model and its use for the valuation of American options.
The multiperiod binomial model: formula for valuing a generic payoff and derivation of the formula for the European call. Choice of u and d parameters and historical volatility.

One-period models
One-period models with an arbitrary number of securities and world states. Payoff matrix.
Replicability, market completeness, characterisation of completeness.
Definition of arbitrage opportunities. Definition of the vector of state prices and first fundamental theorem of valuation. Second fundamental theorem of valuation.
Superreplication and subreplication as a linear programming problem.

Models in continuous time
Definition and first properties of Brownian motion
Ito processes: definition and examples (Brownian motion with drift, geometric Brownian motion)
Ito formula: drift and volatility of a transformed process
Geometric Brownian motion, recalls on lognormal distribution.

The Black-Scholes model
Model assumptions. Derivation of the Black - Scholes differential equation. Special solutions, superposition principle. Derivation of the BS formula as the expected discounted value of the payoff. First properties of the BS formula. Parameter dependence and calculation of greeks. Approximation for short-term ATMF options. First extensions of the BS model: presence of induced flows. Empirical verifications of the BS model. The implied volatility and the smile.

The Merton model
Generalities on credit risk. The Merton model. Calculation of the risk-neutral probability of default. Analytical derivation of the spread rate curve.

The Monte Carlo method
Simulation of random numbers. Calculation of the price of a derivative instrument using the Monte Carlo method.
Delta-hedging.

The VIX Index.

The main Bloomberg functions on options: OMON, OV, OSA, SKEW.

Prerequisites

Some of the basic knowledge of Mathematics, Financial Mathematics and Statistics required for admission to the Master's degree in Economics and Finance. In particular:
- Mathematics: elementary functions, limits, derivatives, integrals, convexity and concavity, vectors, matrices, operations between matrices, linear combinations, linear dependence and independence, rank of a matrix, determinant, inverse matrix, solution of linear systems.
- Financial mathematics: capitalisation and discounting, duration, basic concepts on options (which will however be refreshed at the beginning of the course)
- Statistics: mean, variance, covariance, correlation and their properties. Fundamentals of
probability: sample space, events, probability, random variables. Discrete and continuous random variables, in particular the binomial, the normal, the lognormal and their properties.

Teaching methods

Classes are held in person. Teaching materials are made available to students before the lesson. If a student is unable to attend a class, they can study the materials independently, and if anything is unclear, they can ask questions in the dedicated forums on the e-learning page. Each question receives a response from the professor.

Interactive Teaching
Classes are conducted in two-hour blocks. In each lesson, a variable amount of time between 15 and 45 minutes is dedicated to interactive activities such as:
i) discussion of problems assigned as homework in previous lessons, whose solutions form an integral part of the course
ii) implementations in Excel or Matlab Online using a bring your own device approach for formulas or concepts covered during the lesson
iii) questions posed to students individually or in groups
iv) collective critical use of AI tools such as ChatGPT or Perplexity

In addition to these activities, the instructor organizes a virtual class on Bloomberg terminals, which can be optionally followed by students at their own pace.

Assessment methods

Written exam with open-ended question and optional oral. There are three types of questions:

1. Theoretical questions aimed at testing the learning of the concepts and reasoning explained in the lectures, and more generally the ability to correctly use the technical language of mathematical finance by forming meaningful sentences
2. Exercises in which the concepts and methods explained are applied to cases qualitatively identical to those explained in the lecture or assigned during the course
3. Exercises aimed at testing the students' ability to apply the concepts and methods explained to situations that slightly differ from the cases explained in the lecture or assigned in the assignment.

Textbooks and Reading Materials

- Lecture notes and slides provided by the lecturer.

For further reading:
- J. Hull "Opzioni e futures"
- J. Cox, M. Rubinstein "Option markets"

Semester

Second Semester

Teaching language

Italian. Some of the materials provided are in English as proficiency in the disciplinary language in both Italian and English is very important in this field.

Sustainable Development Goals