

COURSE SYLLABUS

Basics of Actuarial Mathematics

2425-3-E3301M216

Learning objectives

The course covers the main mathematical tools related to life insurance. In particular, it teaches how to use the tools of Financial Mathematics and Probability to solve typical problems such as

- the construction of a mortality model and the determination of life and death probabilities
- the calculation of the fair premium of a policy
- the calculation of the mathematical reserve associated with a policy.

The course also aims to illustrate the links between the concepts of actuarial mathematics and those of expected utility theory and risk theory, which will be briefly recalled.

Contents

- 1) The modelling of the duration of human life: mortality tables and the probabilistic model
- 2) The calculation of actuarial values
- 3) The calculation of the fair insurance premium
- 4) Mathematical reserve, recursive equations, premium decomposition, profit determination
- 5) Expected utility theory and Esscher transforms

Detailed program

- 1) The modelling of the duration of human life: mortality tables and the probabilistic model
Survival function, conditional survival function, mortality force, link between mortality force and survival function, Gompertz's law, Makeham's law, complete and incomplete life expectancy, relationship between complete and incomplete life expectancy
Mortality tables, probability of life, probability of death, deferred probability of death and related actuarial notations.

2) The calculation of actuarial value

Concept of actuarial value, technical bases, calculation of actuarial value for deferred lump sum benefits, temporary mortality covers, temporary mortality covers with variable insured capital, mixed covers, whole life covers, temporary and perpetual life annuities, related actuarial notations. Recursive relationships.

3) The calculation of the fair premium

Definition of equitable premium. Single premiums, periodic premiums, natural premiums. Calculation examples.

4) Mathematical reserve, recursive equations, premium decomposition, profit determination.

Definition of mathematical reserve. Calculation examples. Fourer equation and its interpretation. Decomposition of premium into risk premium and savings premium. Decomposition of profit into financial profit and mortality profit. Homans' formula.

5) Utility theory and general principles of premium calculation.

Recalls on expected utility theory. Definition of indifference premium. Link between indifference premium and fair premium. Recalls on convex functions and Jensen's inequality. Exponential premium and calculation examples. Definition of Esscher transformation and Esscher premium. The case of discrete random variables, the normal, the exponential and the gamma.

Prerequisites

Some concepts of Mathematics, Financial Mathematics and Statistics studied in the propedeutic courses of Matematica Generale, Matematica per la Finanza and Statistica per la Finanza will be used in the course. In particular:

- elementary functions, limits, derivatives, integrals, convexity and concavity
- concept of random variable, distribution function, density function, mean, variance, moment generating function
- discrete and continuous random variables: binomial, Poisson, normal, exponential, gamma
- choice under uncertainty and the notion of expected utility

Teaching methods

Classes are held in person. In case of particular difficulties, they can be streamed.

Teaching materials are made available to students before the lesson.

Interactive Teaching

Lessons are conducted in groups of three consecutive hours. In each lesson, a variable amount of time between 15 and 45 minutes is dedicated to interactive activities such as:

- i) discussing problems assigned at home in previous lessons
- ii) implementing formulas or concepts seen during the lesson in R or Excel using a bring your own device approach
- iii) questions posed individually or in groups to students
- iv) collective critical experimentation with AI tools such as ChatGPT or Perplexity

Assessment methods

The examination is written with open-ended questions and optional oral and is aimed both at verifying knowledge and understanding of the theoretical concepts explained in lectures and at testing the ability to apply the concepts to the solution of simple exercises similar to those assigned and solved independently by the students during the course.

Textbooks and Reading Materials

\ Slides provided by the lecturer

For further information

- Introduction to Insurance Mathematics, A. Olivieri, E. Pitacco, Springer 2011.

Semester

First Semester

Teaching language

Italian

Sustainable Development Goals
