UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

## COURSE SYLLABUS

# Relativity

**2425-3-E3001Q073**

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### Aims

Contents:

- Detailed study of Einstein's special relativity and of some of its main consequences.
- Covariant formulation of the laws of relativistic dynamics and of the electromagnetism (Maxwell equations).
- Lagrangian formalism and introduction to classical field theory, fundamental prerequisite for quantum-field-theory studies.

The student, at the end of the course,

1. will be able to use the mathematical apparatus underlying the "covariant" formulations of special relativity (4-vectors, tensors), both for the theoretical discussion of fundamental concepts and for the resolution of kinematics and electromagnetism problems
2. will be able to connect the non-covariant formulation of electrodynamics with the covariant one
3. will be able to derive the equations of motion for a particle or for a field (scalar or vector) starting from the action and discuss the content and conservation laws of the energy-momentum tensor

### Contents

Einstein's special relativity. Covariant formulation of the relativistic dynamics and of the classical electrodynamics. Relativistically-invariant Lagrangian formalism. Classical field theory: scalar and vector fields.

### Detailed program

**Introduction to Lorentz transformations. Relativistic kinematics. Covariant formulation of Special Relativity (4-vectors, tensors). Lorentz group.**
Refs: Barone, Jackson (Weinberg).

- Quick recap of Classical Mechanics and Electromagnetism (EM) (principle of relativity, Galilean transformations, Maxwell's equations, wave equations). Non-invariance of EM under Galilean transformations, ether hypothesis, Michelson-Morley experiment.
- Bases of Special Relativity: intertial systems, synchronising clocks, postulates, events and intervals, invariance of the speed of light and Lorentz transformations.
- Consequences of Lorentz transformations: time dilation, length contractions, proper time. Minkowski diagrams. Simultaneity, causality. Composition of velocities. Boosts in a generic direction.
- Brief discussion of the more famous "paradoxes" and of physical applications.
- Compact notation for Physics in Euclidean space: vectors, differential operators, various identities. Maxwell's equations (for fields and potentials) in compact notation.
- Special relativity in covariant notation: Minkowski space-time, metric, tensor calculus (covariant and controvariant vectors, tensors, the metric tensor, scalar quantities, differential operators).
- Covariance ("invariance in form") of physical laws and the principle of relativity.
- Lorentz group: general properties, subgroups and classification of homogeneous Lorentz transformations. Generators and algebra of the restricted Lorentz group.
- Relativistic kinematics in covariant notation: 4-velocity, 4-acceleration, energy-momentum 4-vector and its properties. Einstein's relation between energy and mass, 4-momentum conservation.
- Relativistic kinematics: exercises and applications.
- Lorentz boosts in different directions and Thomas precession.

**Relativistic dynamics of a particle; Maxwell's equations in covariant form.**
Refs: Barone, Jackson (Weinberg, Landau)

- Dynamics of a relativistic particle: 4-force and force-acceleration equation.
- Maxwell's equation in covariant form: 4-current, continuity equation, 4-potential, gauge transformations, F?? tensor. Transformation laws of electric and magnetic fields. Invariants of the electromagnetic field.
- Covariant form of the Lorentz force. Interaction of EM fields with charged particles: motion in constant and uniform E and B fields.
- Charged particle with spin in an electromagnetic field.
- "Spin-orbit" interaction of an electron in a central field.
- Bargmann-Michel-Telegdi equation (+).
- Solution of the wave equation in covariant form (+). Radiation by moving charges (+).

**Lagrangian formulation of the electrodynamics. Scalar and vector fields. Stress-energy tensor.**
Refs: Barone, Jackson (Landau).

- Principle of stationary action and lagrangian formulation of the relativistic equations of motion for a free particle and for a charge in an electromagnetic field.
- Classical field theory: introduction and Euler-Lagrange equations.
- Scalar fields and Klein-Gordon equation.
- Stress-energy tensor.
- Vector fields: the Lagrangian of the electromagnetic field (free or interacting).
- The stress-energy tensor for the free and the interacting electromagnetic field.
- Noether's theorem (+).

(+) = advanced topic (covered only time permitting)

## Prerequisites

Classical mechanics, classical electrodynamics, calculus (integration, differential equations, Dirac's delta function). (i.e. the content of the following courses: Fisica 1, Fisica 2, Meccanica Classica and Matematica per la Fisica)

## Teaching form

Frontal teaching (lessons at the blackboard)

## Textbook and teaching resource

**Main textbooks:**
*Relativita'. Principi e Applicazioni*, V. Barone

*Classical Electrodynamics*, J.D. Jackson
Chapter 11: Special Theory of Relativity
Chapter 12: Dynamics of Relativistic Particles and Electromagnetic Fields

**Other useful textbooks:**
*Gravitation and Cosmology*, S. Weinberg
Chapter 2: Special Relativity

*The Classical Theory of Fields (Volume 2)*, L.D. Landau e E.M. Lifshitz
Chapter 1 to 4

*Spacetime Physics*, E.F. Taylor e J.A. Wheeler
Relevant parts available at the teacher's webpage.

- Various notes, complementary material, and exam sheets from previous years are available at the webpage https://virgilio.mib.infn.it/~re

## Semester

First term.

## Assessment method

The exam consists of a written and an oral test.
A sufficient written test is a requirement to be admitted to the oral test. Normally the oral test takes place few days after the written one.

- Written test: exercises on the topics discussed during the course (objective: practical understanding of the fundamental concepts discussed during the course and ability to use the appropriate formalism). At the lecturer's webpage many exam sheets from previous years are available. Several solutions are available too.

- Oral test: discussion on the topics discussed during the course (possibly with a very quick discussion of the written test).

During the academic year, there will be at least 5 exam sessions, in January, February, June, July, Spetember. Typically a sixth session is organised, at an appropriate time, to facilitate students taking the third year exams.

**For erasmus students: if necessary, it is possible to take both the written and the oral exam in English.**

## Office hours

By appointment.

## Sustainable Development Goals

QUALITY EDUCATION