UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Algebra II

2425-2-E3501Q010

Learning objectives

On the basis of the knowledge acquired in the Algebra I course, the course is aimed to a) illustrate further topics in the theory of rings and fields; b) develop the theory of finitely generated modules over principal ideal domains, with applications to abelian groups and linear algebra.

Achievements of a successful attendance of the course include

Knowledge: The knowledge and the understanding of the principle definitions, theorems and results in the theory of rings and their modules, as well as in field theory.
Abilities: The ability to apply this abstract knowledge to concrete problems in algebra.

Contents

Fields, rings and modules

Detailed program

FIELDS
Field extensions: algebraic and transcendental extensions, degree of an extension, the degree formula.
Splitting field of a polynomial.
Finite fields: construction, subfields, automorphisms, ciclicity of their multiplicative group.

RINGS
Complements of ring theory.

The Chinese remainder theorem (for polynomials, for commutative rings).
Partial fraction decomposition of rational functions.
Unique factorization domains and Gauss's lemma.
Localizations of a domain. Local rings.
The ring of formal series with coefficients in a field, with some applications.

MODULES
Modules over a ring and linear algebra. Free modules: bases, rank, universal property. Torsion.
Modules over principal ideal domains: finitely generated modules; equivalence of matrices and reduction to normal form.
Structure theorem for finitely generated modules.
Torsion modules and primary decomposition.
Invariant factors and elementary divisors.
Application to abelian groups: structure theorem for finitely generated abelian groups.
Application to canonical forms for matrices: companion matrix, rational canonical form, Jordan canonical form.

Prerequisites

The contents of the courses Linear Algebra and Geometry, and Algebra I.

Teaching methods

48 hours of frontal lectures and 24 hours of problem classes, delivered in-person.

Assessment methods

Written exam, followed by mandatory oral exam.

The written exam will comprise open questions (not multiple-choice questions) on the theory, and exercises.

The oral examination will be on the theory presented in the lectures, but may include exercises, and possible reference to the text of the written exam.

In both cases the questions will concern definitions, examples, counterexamples, exposition and application of theorems as well as their proofs.

Textbooks and Reading Materials

N. Jacobson, Basic Algebra I, Freeman Co, 1985.

Further literature:

S. Bosch, Algebra, Springer-Verlag, 2003.

B. Hartley; T. Hawkes. Rings, modules and linear algebra, Chapman; Hall 1970

Semester

First semester

Teaching language

Italian

Sustainable Development Goals

QUALITY EDUCATION