UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematical Statistics

2425-3-E3501Q062

Aims

To know and understand the basic tools of inferential statistics.

To be able to apply this knowledge, even using specific software, to situations where there is a sample of observations, in order to provide estimates of unknown parameters and well-founded opinions on the properties of these parameters, after an appropriate choice of the probabilistic model.

Contents

Statistical Models. Inferential Statistics: Estimators, Confidence Intervals, Hypothesis Testing. Use of Specific Software such as R or Spreadsheets.

Detailed program

Introduction to Statistics

Population, random sampling. Sampling problem. Statistics. Densities depending on unknown parameters.

Point Estimates of Parameters

Estimator, unbiased estimator, mean squared error, mean square consistency, necessary and sufficient condition for mean square consistency of a sequence of estimators. Estimators for the moments of a random variable: sample moments. Sample mean. Unbiased estimator of variance: sample variance. Method of moments for constructing estimators. Likelihood function. Maximum likelihood estimator. UMVUE estimator; lower bound of

variance (Cramér-Rao inequality). Invariance properties of maximum likelihood estimators. Asymptotic properties of maximum likelihood estimators. Sampling from normal random variables: law of the sample mean. Law of the square of a normal(0,1): chi-square distribution with 1 degree of freedom. Law of the sum of squares of independent standard normals: chi-square distribution with k degrees of freedom. Law of the sample variance of a normal sample. Student's t-distribution.

Interval Estimates

Confidence intervals: definition, confidence level. Intervals for the mean of a normal population (known or unknown variance). Intervals for the variance of a normal population (known or unknown mean). Pivotal quantity and its use in calculating confidence intervals. Confidence intervals for large samples (particularly for frequencies, i.e., Bernoulli parameters). Pivotal quantity for samples from an absolutely continuous distribution.

Hypothesis Testing

Test for a statistical hypothesis; non-randomized test and critical region. Significance level, p-value. Uniformly most powerful test of a fixed level. Test for the mean of a normal population (known or unknown variance). Test for the variance of a normal population (known or unknown mean). Test for difference of means for normal populations. Comparison between intervals and tests. Test on one frequency and on two frequencies (large sample). Simple and generalized likelihood ratio test. Neyman-Pearson theorem. Pearson's chi-square test for goodness of fit (with or without estimated parameters). Pearson's chi-square test for independence.

Linear Regression

Simple and multiple linear regression: definition, interpretation, tests.

Prerequisites

Mathematical Analysis I and II, with a focus on integral calculus.

Basic Probability: laws of discrete and continuous random variables. Expected value and variance. Law of functions of random variables. Independence. Convergence of sequences of random variables.

Teaching form

A hybrid teaching approach is used, that combines lecture-based teaching (DE) and interactive teaching (DI). DE involves detailed presentation and explanation of theoretical content. DI includes active student participation through exercises and problems, short presentations, group discussions, and group or individual work. It is not possible to precisely determine in advance the number of hours dedicated to DE and DI, as these methods are dynamically intertwined to adapt to the course's needs and promote a participatory and integrated learning environment, combining theory and practice.
The lessons (48 hours, 6 ECTS) are conducted in person and are held in Italian. Upon request and by mutual agreement between the instructor and students, it will be possible to use the English language.
Depending on the availability of recording equipment, the lessons will be recorded and made available on the e-learning website.
Subject to the availability of computer labs, 20% of the course will include laboratory content through the use of statistical software

Textbook and teaching resource

Introduction to Statistics by A.M. Mood, F.A. Graybill, D.C. Boes, 1991, McGraw-Hill Italia, ISBN: 9788838606618

An Introduction to Mathematical Statistics: F. Bijma, M. Jonker, A. van der Vaart, Amsterdam University Press

Other materials: Lecture slides and exercise materials on elearning.unimib.it

Semester

Second semester

Assessment method

Written Exam Structure:

Open-ended questions: These questions cover theoretical concepts described in the course, including requests for definitions and/or statements and proofs of theorems.
Written exercises: These involve applying theoretical concepts and similar techniques to exercises proposed during lectures and for homework.
Presentation: This includes presenting a report on a problem to be analyzed at home.
For Part 1, students are not allowed to use teaching materials. For Part 2, students can use a self-produced formula sheet written on an A4 sheet (front and back), as well as statistical tables for normal, Student's t, and chi-square distributions.

The written exam typically lasts from a minimum of 1 hour and 30 minutes to a maximum of 2 hours and 20 minutes, depending on the length of the questions. Part 1 is evaluated based on the accuracy, clarity, and completeness of the answers. Part 2 is assessed based on the correctness of the procedure and the accuracy of the answers. Part 3 is evaluated based on the clarity and completeness of the presentation.

Parts 1 and 2 contribute equally to 2/5 of the final score each, while Part 3 has a weight of 1/5.

The oral exam is upon request of the teacher and/or the student and consists of a discussion about the written exam, topics covered in class, and the report. In the oral exam, the same qualities of responses evaluated in the written exam are considered. Students with an insufficient but passing grade of 16/30 or higher in the written exam can request to take the oral exam, as well as all other students with a passing grade. If the oral exam is taken, the weight of the written exam is 3/4 and that of the oral exam is 1/4. Generally, passing grades (including the maximum grade) can be confirmed without an oral exam, but the teacher may request an oral exam in all situations where clarification regarding the written exam is deemed necessary.

Office hours

By appointment

Sustainable Development Goals

QUALITY EDUCATION