UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Linear Algebra and Geometry

2425-1-E3501Q051

Aims

In line with the educational objectives of the Degree in Mathematics, the course aims to provide an introduction to linear algebra with applications to geometry, essential to prepare the student to understand the mathematics that will be taught in other courses.
Students are expected to gain knowledge of fundamental notions on vector spaces, diagonalization of endomorphisms and scalar products. They are also expected to gain the ability to reproduce the proofs presented in the course, to solve easy problems using the techniques they have learned, and to delve further, with or without guidance, into some of the results presented during the course.

Contents

Vector spaces; systems of linear equations; linear maps; matrices; diagonalization of an endomorphism; scalar products; affine and euclidean geometry.

Detailed program

- Systems of linear equations: Gaussian elimination method, Rouchè-Capelli Theorem.
- Matrices: matrix product, rank, the ring of square matrices and invertible matrices.
- Vector spaces: generators, basis and dimension; linear subspaces; Grassmann Theorem.
- Linear maps: kernel and image, relation between rank and nullity, matrices associated to linear maps and isomorphisms.
- Determinant of a square matrix and properties; Laplace theorem and Binet theorem.
- Eigenvalues and eigenvectors of an endomorphism; characteristic polynomial of endomorphisms of finite dimensional vector spaces, diagonalization.

- Dual space and dual base.
- Scalar products, orthogonal basis and Sylvester Theorem; Euclidean spaces and Gram-Schmidt process.
- Self-adjoint operators and spectral Theorem.
- Affine spaces, affine coordinate systems, affine subspaces and their representations. Distance and orthogonality.
- Euclidean classification of plane conics.

Prerequisites

Good knowledge of high school mathematics.

Teaching form

The course is organized as follows:
-Lectures (48 hours equal to 6 ECTF) in person;
-Exercises classes (24 hours equal to 2 ECTF) in person.

Both provide lecture-based teaching to deliver the fundamental concepts of Linear Algebra and Geometry..
Definitions, results, and relevant theorems will be discussed in Lectures, providing examples and problems making use of the notions introduced. Exercises on the subject matters covered in the lectures are presented and solved during Exercise classes.

Some exercise sets will be made available regularly on the e-learning website to encourage participation. At the webpage of the course students can find self-assessment quizzes realating to topics covered in the lectures.

A tutor will provide students with support in solving the exercises published on the e-learning website.

The course is delivered in Italian.

Textbook and teaching resource

Reference books:

- M. Abate, Geometria, McGraw Hill, 2002.
- S. Lang, Algebra Lineare, Boringhieri, III edizione.
- E. Schlesinger, Algebra lineare e geometria, Zanichelli 2017

Lecture notes on the e-learning webpage.

Semester

First semester.

Assessment method

Written and oral exams, evaluated on the basis of correctness, completeness, precision, and clarity of the answers.

There are **3 ongoing tests**. The dates will be avaible at the beginning of the course.
In each test students have to upload their homework on line at a fixed deadline. The test will be evaluated on the basis of correcteness, of method and mathematical language, the maximal score is 10. If the total score is at least 18/30, it will give right to a **bonus** (up to 3 points) that concurs to the final evaluation. This bonus expires after the first two exam calls.

- **Written exam.** It consists of two parts:

    1. exercises (with open-ended questions) for evaluating the ability to apply the theoretical results in solving problems;
    2. theoretical questions where the student is asked to answer by giving definitions, statements of theorems, simple proofs of results and provide examples and motivations.

  The examination lasts two hours and half. The maximum score is 32 points: up to 24 for the exercises, and up to 8 for the theoretical question. The passing score of the written exam is 18 points.

- **Oral exam.** Admission to this test is subject to passing the written test. The oral exam begins with the discussion of the written test and it goes on with the request of definitions, theorems, and proofs, in order to evaluate knowledge of contents of the course and ability to rework learned concepts and to expose rationally. In order to pass the exam the oral test must be sufficient.
  The score proposed at the end of the oral test also takes into account both the score of the written part and any possible bonus. This score is the final vote of the exam.

The exam is passed if the final score is at least 18 points.

It is possible to get the **exemption from the oral exam**. Once the written exam has been passed, one has two possibilities:

- to take the oral test;
- to record the vote obtained as follows: the minimum between S and 27, being S the score of the written part increased by the possible bonus.

Note that the bonus does not contribute to pass the written exam.
Note that to record a vote higher to 27 it is necessary to take the oral exam.
The teachers can require an oral exam if the theoretical part of the written test is quite unsatisfactory.

There are 6 exam sessions.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION