UNIVERSITÀ DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Matematica e Statistica

2425-2-E3201Q116

Aims

- To acquire knowledge and understanding of the fundamentals of linear algebra, differential calculus in several real variables, differential equations, and descriptive and inferential statistics.
- To acquire the mathematical and statistical knowledge necessary for the quantitative analysis of environmental data.
- To acquire the ability of independently make judgments in the application of the learned methodologies to the modeling and solving of environmental problems.
- To acquire independence in the application of learned methodologies.
- To acquire the ability to present in a precise and exhaustive way both the learned theoretical knowledge and the independently developed solutions of exercises and problems.
- To be able to understand the modeling/mathematical contents of the courses delivered within the Degree Course.

Contents

- Vectors in R? , matrices.
- Differentiation in Rn.
- Differential equations.
- Descriptive statistics.
- Basic probability.
- Inferential statistics.

Detailed program

- Linear algebra

Vector spaces: sum of vectors, product for a scalar. the vector space R?: inner product, norm of a vector and its properties. Schwarz's inequality, triangle inequality, linear combinations, dependent and independent vectors. Matrices and matrix operations: matrix transpose, matrix addition, product for a scalar and product between matrices. Systems of linear equations and Gaussian elimination method.

- Curves

Vector functions of a real variable, limits, and continuity. Curves, closed curves, simple curves, and planar curves. Support of a curve. Derivative and tangent vector to a curve. Regular and piecewise regular curves.

- Differential calculus for functions of several real variables

Sets in R?. Spherical neighborhoods. Functions of several real variables: introduction and first examples, state functions of thermodinamics. Graphs and level sets. Definition and properties of limits for functions of several variables. Finite limits. Continuous functions. Partial derivatives and gradient, definition of differentiability, link between differentiability and continuity and between differentiability and derivability. Derivability along a given direction and the gradient formula, geometric meaning of the gradient. Sufficient condition for differentiability and the class $C^1(R?, R)$. The first differential. Derivative of the composite function: the case $p(x) = g(f(x))$ with $f: R? \to R$ and $g: R \to R$ and the case $p(t) = f(r(t))$ with $f: R? \to R$ and $r: R \to R?$. Level curves and the gradient. Positively homogeneous functions and Euler's theorem, application to thermodynamic potentials. Higher order derivatives and the Hessian matrix. Schwarz's theorem and C^2 class. Maxwell's relations in thermodynamics. Vector functions of several real variables, Jacobian matrix. General case of the chain rule. Extremal points. Free and constrained extrema. Stationary points (or critical points). Necessary condition for free extrema (Fermat's theorem). Sufficient condition for the two variables case.

- Differential equations

Definition. Ordinary differential equations and partial differential equations with examples. Exponential growth model and logistic model. Order of a differential equation and systems of differential equations. Differential equations in normal form and equivalence with first-order systems. Cauchy's problem. Cauchy's problem for differential equations in normal form of order n. Existence theorem (Peano) and local existence and uniqueness theorem. Differential equations with separable variables and linear differential equations of the first order. Homogeneous and nonhomogeneous linear differential equations of order n. Structure of the general integral of homogeneous and nonhomogeneous equations. Solution of homogeneous linear equations with constant coefficients of order two. Differential equations associated with RLC circuits and damped harmonic oscillator and their general integrals. Solution of a nonhomogeneous linear equation with constant coefficients when the nonhomogeneous term is a polynomial or an exponential (similarity method). Short overview of the qualitative solution of autonomous differential equations: single equations and 2X2 systems. Qualitative analysis of solutions of the following models: logistic equation; logistic equation with extinction and harvesting; Lotka-Volterra predator-prey model; model for two species in competition.

- Descriptive statistics

Absolute and relative frequency tables. Histograms. Sample mean, median, and mode. Sample variance. Sample standard deviation. Quartiles. Interquartile range. Box plot. Bivariate data. Scatter plot. Association between two variables. Sample covariance and sample linear correlation coefficient. Correlation does not imply causation.

- Introduction to probability

Introduction to probability (sample space, events, probability measure). Definition of probability. Properties of probability. Uniform probability. Counting principles (sum rule and product rule). Introduction to conditional probability. Definition and examples with conditional probability. Independence of events. Introduction to random variables. Discrete random variables and discrete density. Expected value and properties for random variables with

a finite number of values. Transfer formula for a random variable. Expected value of the sum of two random variables. Variance and properties. Variance of the sum of two random variables. Covariance and uncorrelated random variables. Uniform discrete and Bernoulli random variables. Continuous random variables and density. Expected value, variance, transfer formulas for continuous random variables. Independence of random variables. Chebyshev's theorem. Distribution function. Uniformly distributed random variables over an interval. Normally distributed random variables and properties.

- Inferential statistics

Introduction to inferential statistics. Unbiased and consistent estimators, with an example of the sample mean through the law of large numbers. Distribution of the sample mean of a normal population. 95% confidence interval for the mean of a normal sample with known variance. Two-tailed and one-tailed confidence intervals at any level for the mean of a normal sample with known variance. Central limit theorem. Two-tailed and one-tailed confidence intervals at any level for a Bernoulli sample with unknown parameter. Hypothesis testing. Critical region. Type I and Type II errors. Significance level and power function. Two-tailed Z-test for the mean of a normal population with known variance. p-value. One-tailed Z-test for the mean of a normal population with known variance. Two-tailed and one-tailed Z-tests for a proportion. T-test for the mean of a normal population with unknown variance. Chi-square goodness of fit test.

Prerequisites

Differential and integral calculus for real functions of a single real variable. Even if it is not formally required, it is necessary to know and to be able to handle the contents of Mathematics I in order to be able to follow the course profitably.

Teaching form

24 lessons of 2 hours each of in-person, lecture-based teaching (6 ECTS)
10 exercise classes of 2 hours each of in-person, lecture-based teaching (2 ECTS)

Course delivered in Italian

Textbook and teaching resource

- Matematica Generale, A. Guerraggio, Bollati Boringhieri. (Linear algebra)
- Analisi Matematica II, M. Bramanti, C.D. Pagani, S. Salsa, ZANICHELLI. (Differential calculus for functions of several real variables and differential equations)
- Probabilità e statistica per l'ingegneria e le scienze, S. M. Ross, Apogeo.
- Esercitazioni di Analisi Matematica 2, M. Bramanti, Esculapio, Bologna. (Exercises)
- Esercizi di calcolo delle probabilità e statistica, D. Bertacchi, M. Bramanti, G. Guerra, Esculapio.
- Probabilità & Statistica 600 esercizi d'esame risolti, M. Verri, Esculapio.

Semester

First semester

Assessment method

The exam consists of a mandatory written test and an optional oral test (upon request of either the student or the instructor), which can be taken if a score of at least 18 is achieved in the written test.

The written test is divided into two parts. The first part consists of 4 open questions, each worth 3 points. In this first part, students will be evaluated on their understanding of the fundamental mathematical and statistical concepts covered during the course. Students need to demonstrate their theoretical knowledge and the ability to explain the key conceptscovered in the course. In the second part, students are required to solve some exercises/problems, usually 4. Each exercise is worth 5 points unless otherwise indicated. In this part of the test, the ability to apply the learned knowledge to solve exercises and problems is evaluated, as well as the ability to communicate the solution process clearly and effectively.
The test must be completed within 120 minutes. To pass the exam, a minimum score of 18 is required.

The optional oral test consists of an interview on the topics covered in the course. Both theoretical understanding and the ability to apply mathematical and statistical concepts to solve concrete problems will be assessed. Based on the performance in the oral test, the score obtained in the written test can be increased by a maximum of 4 points or decreased to a failing grade in case of significant unpreparedness.

Partial exams: During the lecture period, there are typically two partial exams that replace the written test if passed. The partial exams follow the same structure as the written test. The first partial exam covers the program taught until the time of the exam, while the second partial exam covers the remaining program. The partial exams are considered passed if a score of at least 18 is obtained in both exams, and the final grade will be the arithmetic mean of the two scores.

During the exams, consulting educational material (textbooks, exercise books, personal notes, formulas) is not allowed, and the use of cell phones, tablets, PCs, smartwatches, etc. is prohibited. The use of a non-programmable and non-graphing scientific calculator is allowed.

Office hours

By appointment

Sustainable Development Goals

QUALITY EDUCATION