UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

# COURSE SYLLABUS

# Real Analysis and Differential Equations

**2425-1-F4001Q076**

## Aims

According to the Mathematics Degree educational objectives, the course aim is the introduction to linear partial differential equations with hints to nonlinear ones. The skills needed to understand and analyse the most important techniques in the theory and the ability to solve exercises and problems will be provided.

## Contents

Spectral theory for compact and selfadjoint operators. Elliptic equations: regularity, maximum principles, eigenvalues and eigenfunctions of the Laplacian. Transport equation and characteristics. Bochner's integral. Partial differential equations of parabolic type: fundamental solution of the heat equation, Galerkin's method, energy estimates and maximum principle.

## Detailed program

**Second order elliptic equations:** regularity of weak solutions, weak and strong maximum principles.
**Spectral theory**: adjoint, selfadjoint, compact operators, spectrum. Spectrum of compact operators. Fredholm alternative. Spectral decomposition of selfadjoint compact operators. Eigenvalues and eigenfunctions of the Laplacian.
**Transport equation:** The method of characteristics.
**Bochner integral**: Definition, main properties and Sobolev spaces defined with the Bochner integral.
**Parabolic equations:** Fundamental solution for the heat equation and its application. Duhamel's principle. Weak solutions for second order parabolic equations. Galerkin's method. Energy estimates, existence and uniqueness of weak solutions. Maximum principle. Solution to the heat equation through Laplacian eigenfunctions. The Banach fixed point applied to the study of Semilinear parabolic equations.

## Prerequisites

Banach and Hilbert spaces, **L**? spaces, their duals and properties, Sobolev spaces and immersion theorems.

## Teaching form

56 hours of classroom lessons, delivered didactics (8 ECTS credits)

Course delivered in Italian with the possibility of being delivered in English if foreign students request it.

## Textbook and teaching resource

- A. Bressan. Hyperbolic systems of conservation laws: the one-dimensional Cauchy problem. Vol. 20. Oxford University Press on Demand, 2000.
- A. Bressan. Lecture Notes on Functional Analysis. With applications to linear partial differential equations. American Mathematical Society, 2013.
- H. Brezis. Functional analysis, Sobolev spaces and partial differential equations. Springer Science and Business Media, 2010.
- L. C. Evans, Partial Differential Equations, AMS Graduate Studies in Mathematics, Vol.19. Second Edition, Providence 2010.
- D. Gilbarg, N. S. Trudinger, Elliptic partial differential equations of second order, Reprint of the 1998 edition. Classics in Mathematics. Springer-Verlag, Berlin, 2001.

Course webpage: https://elearning.unimib.it/course/view.php?id=44762

## Semester

Second semester.

## Assessment method

The exam consists of a written examination and an oral discussion.

The written examination consists in a short essay. The student is asked to develop two topics out of four proposed in two hours. The first chosen topic must be related to the first part of the course and the second topic to the second part. The exposition must be precise, detailed, exhaustive and consistent with the chosen topics and must contains some of the most significant proofs. The ability to present a selection of significant proofs and, above all, the critical and operational knowledge of the definitions and results presented during the course, also by the illustration of examples and counter-examples are evaluated.

The oral discussion is held a few days after the written examination and consists in a short discussion and correction of the short essay. It verifies the mastery of the chosen topics. No other topics or proofs are asked outside of the two chosen topics.

Final grade out of thirty.

## Office hours

By appointment.

## Sustainable Development Goals