UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Metodi Topologici in Teorie di Campo

2425-1-F4001Q117

Aims

According to the educational objectives of the Course, the taught material aims to provide students with the *basic notions* regarding the definitions and the fundamental results for a geometric and topological approach to the study of classical field theory, with particular emphasis on classical vortex dynamics, ideal magnetohydrodynamics and quantum hydrodynamics. The course aims to provide also the necessary *competences* to understand and use standard techniques and the demonstration methods involved in the theory, as well as the *capabilities* to use them to solve exercises and tackle problems.

The expected outcomes include:

- the knowledge and understanding of the fundamental definitions and statements, as well as of the basic strategies of proof in classical geometric and topological field theory; the knowledge and understanding of some key examples where theory is fully applied;
- the ability to recognize the role that concepts and techniques from a geometric and topological approach play in various areas of mathematics (such as vortex theory, ideal magnetohydrodynamics, quantum fluids), and in the mathematical modelling of physical situations (vortex dynamics, relations between energy and complexity, topological defect production, knotting and linking); skills to apply the basic concepts to the elaboration of practical examples and to the solution of posed questions; the ability to communicate and explain in a clear and precise manner both the theoretical aspects of the course as well as their applications to specific situations, possibly in analogous but different contexts.

Contents

With this course we introduce techniques and topological methods to tackle open problems in classical field theory.

Part I. Fundamentals of Green's potential theory, fluid flows and diffeomorphisms, conservation theorems, Euler's equations, Helmholtz's conservation laws, ideal magnetohydrodynamics, magnetic helicity, dissipative equations.

Part II. Potential theory in multiply-connected domains, elements of knot theory, linking number and self-linking, topological interpretation of helicity, geometric decomposition, magnetic relaxation, Gross-Pitaevskii equation, topological defects in condensates, change of topology by reconnection processes.

Detailed program

The course is divided into two parts, the first being of introductory and general character, the second focused on more specific topics of current research.

Part I. Fundamentals of potential theory in terms of Green's identities, fluid flows and diffeoporphisms, kinematic transport theorem, conservation theorems, decomposition of fluid motion, Euler's equations, vorticity transport equation, Helmholtz's conservation laws, Biot-Savart law, Maxwell's equations, ideal magnetohydrodynamics, magnetic helicity, perfect and non-perfect analogous Euler's flows, Navier-Stokes equations, energy dissipation.

Part II. Kelvin's correction for multiply connected domains, elements of knot theory, fluid dynamic interpretation of Reidemeister moves, inflexional configuration and twist energy, linking number and self-linking, derivation of linking number from magnetic helicity, writhing and total torsion helicity, magnetic energy relaxation, groundstate energy spectra, hydrodynamics interpretation of Gross-Pitaevskii equation, topological defects in condensates, change of topology of flux tubes and physical surfaces due to reconnection processes, knot polynomial invariants, measures of topological complexity.

Prerequisites

Elements of differential geometry of curves and surfaces in three-dimensional space, elements of mechanics of continuum systems, differential operators in mathematical physics and balance laws in physics.

Teaching form

Standard lectures on blackborad taught in English and supported by lecture notes (in English) made available to the students.
A hybrid teaching approach is used, that combines lecture-based teaching (DE) and interactive teaching (DI). DE involves detailed presentation and explanation of theoretical content. DI includes active student participation through exercises and problems, short presentations, group discussions, and group or individual work. It is not possible to precisely determine in advance the number of hours dedicated to DE and DI, as these methods are dynamically intertwined to adapt to the course's needs and promote a participatory and integrated learning environment, combining theory and practice.

Textbook and teaching resource

Lecture notes (in English) made available by the lecturer during the course.

Semester

II semester.

Assessment method

Oral exam (in Italian or English) based on 4 questions taken from a list of questions made available to the student at the end of the course. Specific solutions must reproduce the material presented during the course, including detailed proofs of theorems and statements, complete with explicit computations. The final mark is expressed in 30 units.

The written examination paper must show operational *capability* to tackle and solve the proposed questions by using the acquired *knowledge* and the necessary *competence* to reproduce the topics presented during the course.

Office hours

Upon appointment, to be arranged with the lecturer by email contact: renzo.ricca@unimib.it.

Sustainable Development Goals

QUALITY EDUCATION