UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Differential Topology

2425-1-F4001Q111

Aims

The scope of this course is the continuation of the study of Geometry along the path started in the Laurea Triennale (Bachelor). While it is not a strict prerequisite to the other courses in Geometry, which can be taken independently, it aims to unify and connect the different themes and perspectives developed in them.

Differential topology studies the interplay between the differential structure and the topological properties of smooth manifolds. Differential topology is thus also a natural starting base to explore more abstract aspects of algebraic topology. These techniques also yield a geometric approach to intersection theory.

The expected learning outcomes include the following:

- **Knowledge**: the knowledge and understanding of the basic definitions and statements, as well as of the basic strategies of proof in the theory of differential topology; the knowledge and understanding of some of the most relevant basic applications and examples of the theory;
- **competence**: the ability to apply the acquired abstract knowledge to the construction and discussion of simple examples and solution of exercises; the ability to expose and communicate effectively and clearly the theoretical content of the course.

Contents

Transversality and intersection theory.
De Rham Theory for smooth manifolds.

Detailed program

- Transversal maps, intersection of transversal smooth manifolds.
- Transverality for manifolds with boundary.
- Applications: smooth retractions of manifolds and Brower fixed-point Theorem.
- Transversality Theorems, homotopy and families.
- Intersection Theory mod 2.
- Intersection Theory for oriented manifolds: intersection number for oriented varieties, degree of a map.
- Applications: winding numbers and Jordan-Brouwer separation Theorem.
- Lefschtez fixed-point Theory.
- De Rham cohomology groups on smooth manifolds.
- Mayer-Vietoris sequence.
- Poincarè duality on orientable manifolds.
- The Kunneth Theorem.
- De Rham's Theorem.

Prerequisites

The content of the courses of Analysis I, Linear Algebra and Geometry, Geometry I. The basics on differential varieties and differential forms (as content of Geometry II and Geometry III). Brief recalls will be offered as needed.

Teaching form

The course provides lectures (56 hours equal to 8 ECTS) in person.

Some exercises will be proposed, their solutions can be discussed either in class or during office hours.
The course is provided in Italian, but it could be conducted in English in the presence of foreign students.

Textbook and teaching resource

Reference texts:

- V. Guillemin, P. Haine, Differential forms, World Scientific Publishing Co.
- V. Guillemin e A. Pollack, Differential Topology, Prentice Hall
- J.W. Milnor, Topology from the Differentiable Point of View; University Press of Virginia.

Other material:
Lecture notes.

Semester

Second semester

Assessment method

The assessment method consists in **oral exam.**
There are no partial texts.
The exam consists of two parts:
-in the first one there are theorethical questions involving definitions, statements of theorems, proofs,
-in the second one there are computational questions as construction of examples and counterexamples and exercises (similar to those proposed at lectureres).
The two parts will contribute equally to the the final grade (up to 30/30). In the first part, the evaluation will take into account the knowledge and the understanding of conceptual framework of the course, as well the ability to expose it in well-organized manner. In the second part, the evaluation will take into account the ability to perform exercises, the exactness of the answers and the mathematical language used.
In order to successfully complete the exam the students need to obtained a grade of at least 18/30.
There will be 6 exam sessions (june, july, september, november, january and february).

Office hours

Upon appointment.

Sustainable Development Goals

QUALITY EDUCATION