



UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Teoria Geometrica dei Gruppi

2425-1-F4001Q082

Aims

The main goal of this course is the study of certain graphs called trees and of the groups acting on them. This theory was developed by Hyman Bass and Jean-Pierre Serre in the '70s and is now a building block in the study of Geometric Group Theory, which aims to recover group properties by observing how groups act on certain topological spaces. Time permitting, we will see some application of the theory, such as ends of groups, the characterization of subgroups of free groups via Stallings graphs, the boundary of a tree, etc.

- *knowledge*: learning the language, definitions and statements of the main results of Bass-Serre theory;
- *expertise*: concrete understanding of the main proof techniques;
- *skills*: ability to apply the theory to solve problems

The course can be taught in English or Italian. For teaching and additional purposes, the English language would be preferable, but in any case the language will be discussed and decided in person during the first classes of the course.

Contents

- Basics in graph theory and trees
- Construction of Cayley graphs from groups
- Construction of free groups, free products and generalizations
- Characterization of groups acting on trees

Detailed program

- Graphs, paths, connectivity, trees
- Group actions on graphs, Cayley graphs, quotient graphs
- Free groups, diamond and ping-pong lemmas
- Free products (with amalgamation) and HNN-extensions and their realizations through groups acting on trees
- Graph of groups; the fundamental group of a graph of groups
- Characterization theorem for groups acting on trees (the fundamental theorem of Bass-Serre theory)
- Kurosh theorem for subgroups of free products
- Possible additional advanced topics at the end of the course (ends of groups, Stallings graphs, boundary of a tree, etc.)

Prerequisites

Algebra I, Geometria I.

Teaching form

56 hours of in-person, lecture-based teaching (8 ECTS)

Textbook and teaching resource

- O. Bogopolski, Introduction to Group Theory, EMS Textbooks in Mathematics, 2008.
- J. Meier, Groups, Graphs and Trees, London Mathematical Society, Student Texts, 73, CUP, 2008.
- J-P. Serre, Trees, Springer-Verlag, Berlin, 1980.
- W. Dicks, M. Dunwoody. Groups acting on graphs. Cambridge Studies in Advanced Mathematics, 17. Cambridge University Press, Cambridge, 1989
- G. Baumslag, Topics in combinatorial group theory. Lectures in Mathematics ETH Zürich. Birkhäuser Verlag, Basel, 1993

Semester

1st semester

Assessment method

The final exam will consist of a conversation in two parts:

(1) The first part is given by a 20-minute presentation about an application of or a topic related to Bass-Serre theory previously agreed with the instructor. The talk will contribute 20% to the final grade

(2) The second part is an oral discussion through questions about the content covered during class time to assess the students' acquired proficiency in explaining and applying Bass-Serre theory. The oral discussion exam will

provide the remaining 80% of the final grade.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION
