UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Matematica Numerica per il Machine Learning

2425-1-F4001Q118

Aims

In line with the educational objectives of the Master Degree in Mathematics, the course aims to provide the knowledge of the mathematical and numerical methodologies and (available) theories underlying some machine learning techniques.

The techniques and algorithms will be implemented in Python and/or MATLAB, and with the developed codes the students will have the ability to implement elementary algorithms of Machine Learning and to comprehend how more advanced algorithms work.

Contents

- Mathematical Foundations of Learning Problems & Statistical Learning. Description of Classification problems
- Algorithms/methods for Least Squares (general). Statistical Analysis of LS. Bias-Variance trade off
- Kernel Methods
  -Optimization: Gradient Descent & Stochastic Gradient Descent & Subgradient Methods
  -Error splitting and Analysis for Learning problems. Curse of dimensionality
- Neural Networks
- Neural Network Learning and Training

Further topics that will be developed (depending on the students interest and :
-Universal approximation for Neural Nets (2-hidden layer architecture)

- NN to approximate PDEs (PINNs methods)
  -NN to approximate inverse problems and/or to estimate parameters
  -Rigorous Analysis of error estimates in Learning problems

Detailed program

Mathematical Foundations of "General" Regression Problems

- Review of probability basics;
- Model assessment and selection: Empirical Minimization, Hypothesis Space, Bias-Variance Tradeoff;
- Some Numerical Linear Algebra Tools: Singular value decomposition (SVD) and low rank approximation;
  -Algorithms for Least Squares (LS) (general). Statistical Analysis of LS
- Kernel Methods. Reproducing Kernel Hilbert Spaces (RKHS).

Optimization

Gradient Descent & Stochastic Gradient Descent & Subgradient Methods

Neural Networks

- Motivation and Definition. Mathematical Representation (Neurons, Artificial Neural Networks, Artificial Feedforward Neural Networks)

Neural Network Learning and Training

- Neural Network Learning: Motivation, Regression/Classification;
- back propagation;
- Going Deep: Deep Learning. Pros and Cons. Regularization, Convolutional Neural Networks.

*__Further topics__

- Universal Approximation properties of NN
- NN to approximate PDEs (PINNS methods)
  -Rigorous Analysis of Error Estimates in Learning problems

CS Laboratory:
Implementation in MATLAB and/or Python of:

- Feed Forward Neural Networks;
- Back Propagation Algorithm;
- Stochastic gradient descent
  Applications to artificial and natural images recognition.
  Use of MATLAB and/or Python toolbox for:
- Convolutional Neural Networks

Prerequisites

Basic courses of the Laurea Triennale.

Teaching form

Lectures (face to face) and CS Laboratory (8 CFU)

Textbook and teaching resource

The teaching material will be made available by the instructors during the course.

Semester

2?? semester

Assessment method

The evaluation of the course has two parts:

1- the development of a small project
2- an oral exam.

Mark is out of thirty. The student need to reach at least 18/30 in both parts to pass the exam. the final mark is the average of the two partial marks.

The aim of the project is to validate the knowledge and capabilities of the students to use the theoretical and practical instruments developed during the course. Students are encouraged to work on the project **in groups** of at most three people.

The oral exam will evaluate the knowledge and **understanding** of the results and rigorous proofs developed in the course and the capacity to **comprehend** how the algorithms work.

There will be 5 exam sessions (in June, July, September, January, February).

Students who will adhere to the blended modality of the course (by preparing and explaining a specific topic or writing up the scribe notes) will get automatically 5 points more for the final vote.

Office hours

By appointment

Sustainable Development Goals

QUALITY EDUCATION