UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Calculus

2526-1-E3004Q001

Aims

The aim of the course is to provide, both from a conceptual and from a calculus point of view, the basic mathematical tools which are essential to successfully attend a university undergraduate program in a scientific area. The course should also provide the required mathematics prerequisites for the other courses of the study plan.

1. Knowledge and understanding
   By the end of the course, the student will have acquired a solid understanding of the main properties of sets, of the main numerical sets (real numbers in particular), of functions between sets, of elementary functions and of complex numbers. Also, they should know the basic results in the theory of differential and integral calculus for functions of one real variable and of numerical sequences and series. The student should also learn the most important results in the theory of ordinary differential equations and the main integration techniques for linear equations and for some simple types of nonlinear ones.
2. Applying knowledge and understanding
   The student will be able to compute limits of sequences and functions (and, in particular, to manage indeterminate forms), to compute derivatives, to apply the main tools of differential calculus for functions of one real variable. Finally, they will be able to compute integrals by the methods of integrations by parts and by substitution, and to solve some simple types of ordinary differential equations.
3. Making judgements
   The student will develop the ability to recognize which mathematical tools are appropriate for solving specific problems in one-variable calculus, and to assess the validity of assumptions and approximations involved in their application.
4. Communication skills
   The student will be able to express mathematical reasoning with clarity and precision, both in written form and orally. They will be capable of explaining the logical steps behind a calculation or proof, and of interpreting mathematical results in physical terms.
5. Learning skills
   The student will develop the ability to learn new mathematical concepts independently and to apply them in

different scientific contexts. The course will provide the foundations necessary for subsequent more advanced study in physics and mathematics.

Contents

ˆ Numerical sets; real and complex numbers;
ˆ Basic abstract concepts about functions (domain, injectivity, inverse function, composition, etc.);
ˆ Elementary functions: power functions, trigonometric functions, exponentials, logarithms, their graphs and their basic properties;
ˆ Limits of numerical sequences and of functions of one real variable; continuity; main local and global properties of continuous functions;
ˆ Derivatives; main theorems of differential calculus in one real variable;
ˆ Applications of differential calculus; Taylor expansions;
ˆ Definite and indefinite integrals; improper integrals; basic integration techniques;
ˆ Numerical series; convergence criteria;
ˆ Ordinary differential equations and basic resolution formulas.

Detailed program

1. The set of real numbers. Maximum, minimum, supremum, infimum. Elementary properties of functions. Elementary functions. Complex numbers.
2. Limits of sequences. Definitions and first properties. Bounded sequences. Operations with limits. Comparison theorems. Monotone sequences. Undetermined forms. Special limits.
3. Limits of functions and continuous functions. Definition and first properties of limits of functions and of continuous functions. Types of discontinuities. Limits and continuity of the composition of functions. Some important theorems about local and global properties of continuous functions.
4. Derivatives and study of functions. Definition of derivatives. Computation of derivatives. Theorems of Fermat, Rolle, Lagrange and Cauchy and their consequences. Second and higher order derivatives. Monotone and convex functions. Extrema and inflection points. Applications to the study of functions. De L'Hôpital's theorem and Taylor's formula.
5. Integration. Definite integrals and method of exhaustion. Definition of integrable functions and classes of integrable functions. Properties of the definite integrals. Indefinite integrals. Fundamental theorem of integral calculus. Integration methods. Integration by parts and by substitution. Integration of rational functions. Improper integrals and convergence criteria.
6. Numerical series. Definition of numerical series and of their character. Harmonic and geometric series. Convergence criteria for series.
7. Differential equations. Some theoretical facts about ordinary differential equations (Cauchy's problem, global and local solutions, uniqueness, regularity). Integration formulas for linear equations and for some simple types of nonlinear equations.

Prerequisites

The course will require the knowledge of the mathematical notions generally developed in the secondary school (on the other hand, no previous knowledge of mathematical analysis is necessary). Essential prerequisites can be considered the following ones: algebraic equations and inequalities of the first and second degree, planar analytic geometry, trigonometry, exponential and

logarithmic functions. The students who experience some lack of basic mathematical notions from the high school are especially invited to follow the tutoring classes. Moreover, they can take advantage of the MOOC Mathematics pre-course offered by the University of Pavia.

Teaching form

The class schedule will be organized into (indicatively) eight hours per week. Of these, four will be offered in the form of recorded lectures (podcasts), while the remaining four will consist of in-person lectures taking place at the Department of Physics of Pavia University. At the beginning of every week, indicatively four hours of podcasts will be published on the Kiro website of the course (and they will remain available for the rest of the year). Some additional material (in particular, a number of proposed exercises) may be also published weekly. During the in-person classes the teacher will develop additional notions and explanations complementing the material contained in the podcasts. Moreover, some of the proposed exercises will also be discussed.
A tutoring program will also be offered: the teachers or tutors will be available to discuss exercises, to answer questions, and to clarify doubts. Participation to this activity is strongly encouraged.

Textbook and teaching resource

Suggested text:
C. Canuto and A. Tabacco, Mathematical Analysis 1, Pearson, 2022,
also available in Italian:
C. Canuto and A. Tabacco, Analisi Matematica 1, Pearson, 2021.

Semester

First semester

Assessment method

The exam consists of a compulsory written test, possibly complemented by an oral part. The written test is a closed books test: notes, books, calculators or similar instruments, items with a photocamera or able to connect to the internet are not allowed. Students are required to provide an ID card with a photograph.
In the written test, students should solve some exercises on the topics of the course and answer some questions of theoretical character on the program of the course. For some of the exercises or questions, only the solutions or the answers will be required, without any detailed explanation. Other exercises or questions will require a fully detailed solution or answer. For these exercises, both the correctness of the answer and the justification of it are evaluated.
The final mark of the Calculus exam will be obtained as a weighted average of the grades obtained in the exams of Calculus I and Calculus II.

Office hours

By appointment. Please contact the teacher by email only.

Sustainable Development Goals

QUALITY EDUCATION