

# UNIVERSITÀ DEGLI STUDI DI MILANO-BICOCCA

### **COURSE SYLLABUS**

## **Geometry III**

2526-3-E3501Q055

#### **Aims**

This course aims to yield an introduction to the fundamental group and differentiable manifolds.

Expected learning outcomes include:

- (1) knowledge and comprehension: knowledge of definitions and basic results on the fundamental group and differentiable manifolds, and the knowledge of relevant examples. Comprehension of the main proof techniques.
- (2) applying knowledge: ability to solve problems and discuss the geometric properties of concrete examples.
- (3) autonomy: ability to find mistakes in proofs, ability to connect the content of the course with others subjects.
- (4) communication: ability to express clearly and rigorously notions and definitions.
- (5) learning skills: understading the correct approach to study the subject in order to apply the knowledge of the course to future problems.

#### **Contents**

Fundamental group and covering spaces of a topological space. Differentiable manifolds: topological manifolds, differentiable structures, tangent space, tangent bundle, calculus on manifolds. Immersions, submanifolds. Vector fields, flows, Lie bracket. Differential forms, Stokes' theorem, De Rham cohomology.

#### **Detailed program**

The course is divided into 3 modules of 16 hours.

1: Fundamental group and coverings

Definition of the fundamental groupoid and of the (based) fundamental group. Induced homomorphism by a continuous function. Basepoint dependence. Homotopically equivalent spaces and invariance of the fundamental group. Contractible spaces. 1-connected spaces. Coverings. Lifting theorem for continuous functions. Lifting of paths and homotopies. Fundamental group of the circle. Equivalence of coverings. Correspondence between coverings and subgroups of the fundamental group. Universal cover. Seifert Van-Kampen Theorem and presentation of groups. Properly discontinuous actions.

#### II: Smooth manifolds and vector fields

Definition of topological manifold and smooth manifold. Smooth functions. Tangent space to a smooth manifold. Derivation at a point and curves. The differential of a smooth map. Regular and critical values of a smooth map. Immersions and embeddings. Submanifolds of a smooth manifold. Regular value theorem and constant rank theorem. Lie groups and subgroups. Tangent bundle to a smooth manifold. Vector fields and derivations. Lie bracket. Completeness of a vector field. Integral curve and Cauchy type theorem. Flow of a vector field. Invariant vector fields on a Lie group. The Lie algebra of a Lie group.

#### III: Differential forms

Cotangent bundle and exterior powers. Differential forms. De Rham cohomology: closed and exact forms. Orientation on a smooth manifold. Integration of compactly supported differential forms. Partition of unity. Stokes' theorem. De Rham cohomology in top degree. The degree of a smooth map. The degree formula. Invariance of De Rham cohomology and Poincaré lemma. Exact sequences of vector spaces. The exact sequence of Mayer Vietoris. Poincaré duality.

#### **Prerequisites**

The content of the courses of Geometry I and II, Mathematical Analysis I and II, Linear Algebra and Geometry, Algebra I.

#### **Teaching form**

Blackboard lectures (6 ECTS, 48 hours), in class. In italian.

#### **Textbook and teaching resource**

#### **Textbooks**

For the first module:

- 1. G. Bredon Topology and Geometry (Capitolo 3)
- 2. J. Munkres Topology (Capitolo 9-11-13)
- 3. A. Hatcher Introduction to algebraic topology (Capitolo 3)

For the second and the third modules:

- 1. L. Tu An introduction to Manifolds.
- 2. J. Lee Introduction to Smooth Manifolds.
- 3. R. Bott, L.Tu Differential forms in algebraic topology. (Capitolo 1).

Semester
First semester.
Assessment method
Written examination: 4 exercises with a maximum of 28 points. You need 14 points to do the oral examination.  Oral examination: 2 questions about the main topics of the course.  **Both examinations are compulsory. **  There are no mid-term exams.
Office hours
By appointment.
Sustainable Davalenment Coole
Sustainable Development Goals
QUALITY EDUCATION

There are also the notes of the course.