UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematical Analysis III

2526-3-E3501Q056

Aims

The course aims to provide students with the foundational knowledge required to study problems in advanced mathematical analysis. It will also equip them with the skills necessary to understand proof techniques in order to solve exercises and tackle problems in mathematical analysis.

More specifically, according to the Dublin Descriptors, the learning objectives are as follows:

(1) Knowledge and understanding
Students will acquire foundational knowledge in advanced mathematical analysis, with particular focus on Banach and Hilbert spaces, L^p spaces, convolution, linear operators between normed spaces, Baire's theorem and its consequences, Fourier series, and the Fourier transform.
This content builds upon knowledge gained in previous courses and lays the groundwork for further advanced studies. Theoretical understanding will be enhanced through in-depth analysis of topics as well as the study of examples and applications.

(2) Applying knowledge and understanding
Students will be able to apply the acquired knowledge to solve problems in mathematical analysis. Throughout the course, students will be expected to prove statements in analysis using proof techniques similar to those employed in the main theorems covered in the syllabus.

(3) Making judgements
The course aims to develop the student's ability to critically analyze mathematical statements and proofs, with particular attention to how assumptions contribute to the logical structure of a proof. Students will also be encouraged to independently select the most appropriate problem-solving strategies depending on the context. These skills will be further developed through the comparison of different methods for solving the same problem.

(4) Communication skills
Students will acquire the ability to clearly and rigorously communicate mathematical concepts and to present proofs in a structured and comprehensible manner. The use of formal mathematical language will be encouraged, while

also emphasizing the importance of expressing ideas in more intuitive and accessible terms when appropriate.

(5) Learning skills
The course is designed to provide students with the tools necessary to continue their study of mathematical analysis independently at more advanced levels. Students will be encouraged to approach new topics methodically and rigorously, building upon prior knowledge. They will also be stimulated to consult a variety of sources to deepen and update their understanding.

Contents

Banach Spaces. Lp spaces. Hilbert spaces. Fourier series. Convolution. Fourier transform. Baire's Theorem. Open mapping Theorem. Banach Steinhaus Theorem. Dual space. weak convergence.

Detailed program

Definition of Banach space. Examples.
Definition of L ^ p (X, ?), ? positive measure.
Holder and Minkowski inequalities.
Completeness of L ^ p (X, ?).
Inclusions of spaces L ^ p (X, ?), finite ?.
Inclusions of spaces L ^ p (Z).
Relations between pointwise convergence, convergence in Lp, and in measure.
Density of Cc (Rn), Coo (Rn) and of the Schwartz space in L p (Rn).
Duality of Lp spaces (only statement).
Hilbert spaces.
Inner product.
Cauchy-Schwarz Inequality.
Hilbert space.
Points of minimum distance from a closed convex.
Projection theorem.
Bessel inequality.
Complete orthonormal systems.
Parseval formula.
Gramschmidt process.
Fourier series for functions on the thorus
Dirichlet kernel.
Convergence in L2.
Pointwise convergence.
Linear operators between normed vector spaces.
Dual space.
Baire's theorem.
The Banach-Steinhaus Theorem.
Divergence of the Fourier series.
Open Mapping Theorem.
Closed Graph Theorem.
Non surjectivity of the Fourier transform from L 1 (T) into c_0 (Z).
Weak convergence.
Fourier transform in Rn.

Prerequisites

Topology. Linear algebra. Differential calculus. Integral calculus. Measure theory. Complex numbers.

Teaching form

48 hours of in-person, lecture-based teaching (6 ECTS)
Course delivered in Italian.

Textbook and teaching resource

G.B. Folland "Real Analysis"
L. Grafakos "Classical Fourier Analysis"
W. Rudin "Real and Complex Analysis"
W. Rudin "Functional Analysis"
E.M. Stein R. Shakarchi "Functional Analysis"
E.M. Stein R. Shakarchi "Fourier Analysis"

Notes

Semester

Second semester

Assessment method

The exam consists of a written test and an oral test.

Optional ongoing assessments.
During the course, some topics will be proposed for in-depth study in groups of 3-4 students. Each group will be required to write a short paper (a few pages) and present the work orally, with an individual presentation lasting approximately 10 minutes per person. The project will be graded with a maximum of 4 points for the written content and up to 2 points for the oral presentation.
In this way, each student can earn up to 6 points to be added to their written exam score, provided that the written exam grade is at least 12.

Written test

The written test consists of exercises aimed at verifying the understanding of the course content, the ability to apply the learned proof techniques to problem-solving, and clarity of exposition. Each exercise will be assigned a maximum partial score based on its difficulty and length; the student’s score will be determined according to

accuracy, completeness, rigor, clarity, and coherence of the solution. The maximum score for the written test is 33.

The exercises proposed are consistent with those practiced during the lessons.

Admission to the oral exam requires a written test score of at least 16.

The duration of the written test is two hours.

Oral test

The oral exam consists of a discussion of the written test and theoretical questions (definitions and theorems with proofs) on the topics covered in class. The oral exam will assess the knowledge and understanding of the course content, as well as the ability to organize a clear, effective, and well-structured coherent presentation.

The final grade is given by the written test score, to which points may be added or subtracted with the oral exam.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION