UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Analisi Complessa

2526-3-E3501Q057

Aims

Learning Objectives (Dublin Descriptors)

1. Knowledge and Understanding
   The student will acquire a clear and systematic understanding of the main concepts of complex analysis in one variable: holomorphic functions, Cauchy's theorem and its applications, isolated singularities and zeros, conformal mappings, harmonic functions, power series and infinite products, the Gamma function, and the Zeta function.

2. Applying Knowledge and Understanding
   The student will be able to apply the learned methods to solve exercises and problems, including in simple applied contexts, demonstrating mastery of computational techniques and an understanding of basic mathematical structures.

3. Making Judgements
   The student will develop the ability to critically understand and evaluate definitions, theorems, and proofs, identifying the most appropriate conceptual tools for analyzing and solving the proposed problems.

4. Communication Skills
   The student will be able to present the fundamental concepts of the course clearly and rigorously, using mathematical language correctly.

5. Learning Skills
   The student will develop the skills necessary to continue studying complex analysis and related subjects independently, with the ability to consult scientific texts and appropriate educational resources.

Contents

This is a basic course in one complex variable. It includes holomorphic functions, conformal maps, power series, Cauchy's theorem and applications, isolated singularities, zeroes of entire functions and applications. We shall also provide an introduction to some important special functions.

Detailed program

Ecco la traduzione in inglese del testo fornito:

Part 1. Preliminaries.
Holomorphic functions: definition and examples. Entire functions. Cauchy–Riemann equations. Harmonic functions. Power series. Hadamard's formula for the radius of convergence of a power series. Maclaurin series for main elementary functions. Integration along curves. Parametric curves, piecewise regular parametric curves. Orientation. Integration along curves and its properties.

Part 2. Cauchy's Theorem and Applications.
Goursat's lemma. Existence of local primitives and Cauchy's theorem for a disk. Existence of primitives of a holomorphic function in a disk. Cauchy's theorem in a disk. Computation of certain integrals. Examples of computing integrals using Cauchy's theorem. Cauchy integral formula. Maximum modulus theorem and Schwarz lemma. Cauchy inequalities. Holomorphic functions as local sums of power series. Liouville's theorem. Fundamental theorem of algebra. Identity principle for holomorphic functions and analytic continuation. Further applications. Morera's theorem. Uniform convergence on compact sets of sequences of holomorphic functions. Holomorphic functions defined via integrals. The symmetry principle and Schwarz reflection principle. Polynomial approximation problem and Runge's theorem.

Part 3. Meromorphic Functions and the Logarithm.
Zeros and poles. Local form of a holomorphic function near a zero. Multiplicity of a zero, simple zeros. Poles of holomorphic functions. Local form near a pole. Order of a pole, principal part, and residue. Residue formula for a pole of order n. Residue formula. Residue theorem. Examples of applications of the residue theorem. Singularities and meromorphic functions. Removable singularities. Riemann's theorem on removable singularities. Characterization of poles. Essential singularities. Behavior near essential singularities: Casorati–Weierstrass theorem. Meromorphic functions in a region. Singularities at infinity. Characterization of meromorphic functions on the extended complex plane. Argument principle and applications. Argument principle. Rouché's theorem. Open mapping theorem. The complex logarithm. Existence of the logarithm in a simply connected region. Principal branch of the logarithm. Power series expansion of the logarithm. Existence of the logarithm of a non-vanishing function in a simply connected region.

Part 4. Entire Functions.
Jensen's formula. Jensen's theorem. Functions of finite order. Order of an entire function. Relationship between the order of an entire function and its zeros. Infinite products. Definition of convergence of an infinite product. Sufficient condition for convergence. Convergence of products of holomorphic functions. Product formula for the sine function. Weierstrass infinite products. Existence of entire functions with prescribed zeros. Hadamard's factorization theorem. Factorization of entire functions of finite order.

Part 5. Euler's Gamma Function and Its Properties.
Riemann's Zeta Function: its analytic continuation and connection with the Prime Number Theorem.

Prerequisites

The prerequisites are included in the programme of the courses Analisi I, Analisi II, Algebra lineare and Teoria della misura of the Laurea triennale in Matematica. Specifically we require a sound knowledge of differential and integral calculus in one and several variables, basic notions in Linear algebra and a good understanding of the Lebesgue integral, in particular of the Lebesgue dominated convergence Theorem and the Fubini-Tonelli Theorem.

Students lacking prerequisites are invited to contact the professor by e-mail. He will give them bibliographical suggestions useful to fill the gaps and possibly provide further support.

Teaching form

48 hours of in-person, lecture-based teaching (6 ECTS)
Course delivered in Italian.

Textbook and teaching resource

The instructor recommends the following classic texts:

- Ahlfors, *Complex Analysis*, McGraw-Hill
- Nevanlinna, Patero, *Introduction to Complex Analysis*, Chelsea Publishing

and the more recent ones:

- Stein and Shakarchi, *Complex Analysis*, Princeton University Press
- Ullrich, *Complex Made Simple*, American Mathematical Society

The instructor will also provide their own lecture notes and exercises on the course's e-learning platform.

Semester

First semester

Assessment method

No midterm exams are scheduled. The final assessment consists of a written exam and an oral exam. The written exam, which includes exercises and problems requiring application of the theory, grants access to the oral exam if passed. During the oral exam, the student must demonstrate knowledge of the fundamental aspects of the theory, including the statements and proofs of the main theorems.

The exam is considered passed by those who show they possess the required theoretical knowledge and the necessary skills to solve the proposed exercises.

The final grade will take into account the accuracy of the answers, the clarity of exposition, and the appropriate use of mathematical language.

Office hours

By appointment (requested through e-mail)

Sustainable Development Goals

QUALITY EDUCATION