UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematical Statistics

2526-3-E3501Q062

Aims

Knowledge and understanding
The student will acquire a solid understanding of the main concepts of inferential statistics, with particular focus on estimator theory, Fisher information, confidence intervals, and hypothesis testing. The course will deepen the formal framework of statistical models and the theoretical foundations of estimation and inference, promoting a rigorous mathematical comprehension of statistical methods.

Applying knowledge and understanding
The student will be able to apply statistical techniques critically to real-world problems, selecting appropriate models and using computational tools (especially the R software) for data analysis. They will also be able to evaluate the efficiency and properties of estimators within specific models.

Making judgements
The student will develop the ability to autonomously analyze inferential problems, critically assessing model assumptions and the implications of methodological choices. They will be able to compare different inferential methods and justify their decisions in both theoretical and applied contexts.

Communication skills
The student will be able to clearly and rigorously present theoretical and applied topics in mathematical statistics, using appropriate technical language. They will be able to write reports on statistical data analysis and effectively communicate results to both specialists and non-specialists.

Learning skills
The student will develop the skills necessary for independent study of advanced texts in statistics and for deepening theoretical and applied aspects of the field, also in view of pursuing graduate studies or engaging in research activities.

Contents

Statistical Models. Inferential Statistics: Estimators, Confidence Intervals, Hypothesis Testing. Fisher information. Regression Use of Software R.

Detailed program

Random sample.
Statistics.
Densities depending on unknown parameters.
Point estimators of parameters.

Estimator, unbiased estimator, mean squared error, mean square consistency, necessary and sufficient condition for mean square consistency of a sequence of estimators.
Estimators for the moments of a random variable: sample moments. Sample mean.
Unbiased estimator of variance: sample variance.
Method of moments for the construction of estimators.
Sampling from normal random variables: distribution of the sample mean.
Distribution of the square of a standard normal variable: chi-squared distribution with 1 degree of freedom.
Distribution of the sum of squares of independent standard normals: chi-squared distribution with
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k degrees of freedom.
Distribution of the sample variance from a normal sample.
Student's t-distribution.

Confidence intervals: definition, confidence level.
Confidence intervals for the mean of a normal population (with known or unknown variance).
Confidence intervals for the variance of a normal population (with known or unknown mean).
Confidence intervals for large samples (in particular for proportions, i.e. Bernoulli parameters).

Hypothesis testing
Test for a statistical hypothesis; non-randomized test and critical region.
Significance level, p-value. Uniformly most powerful test at a fixed level.
Tests for the mean of a normal population (with known or unknown variance).
Tests for the variance of a normal population (with known or unknown mean).
Tests for the difference of means from normal populations.
Test on a proportion (large sample).
Simple and generalized likelihood ratio tests. Neyman–Pearson lemma.

Linear regression
Simple and multiple linear regression: definition, interpretation, testing.

Prerequisites

Mathematical Analysis I and II, with a focus on integral calculus.

Basic Probability: laws of discrete and continuous random variables. Expected value and variance. Law of functions of random variables. Independence. Convergence of sequences of random variables.

Teaching form

A hybrid teaching approach is used, that combines lecture-based teaching (DE) and interactive teaching (DI). DE (80%) involves detailed presentation and explanation of theoretical content. DI includes active student participation through exercises and problems, short presentations, group discussions, and group or individual work.
The lessons (48 hours, 6 ECTS) are conducted in person and are held in Italian. Upon request and by mutual agreement between the instructor and students, it will be possible to use the English language.
Depending on the availability of recording equipment, the lessons will be recorded and made available on the e-learning website.
Subject to the availability of computer labs, 20% of the course will include laboratory content through the use of statistical software

Textbook and teaching resource

Lecture notes

Introduction to Statistics by A.M. Mood, F.A. Graybill, D.C. Boes, 1991, McGraw-Hill Italia, ISBN: 9788838606618

An Introduction to Mathematical Statistics: F. Bijma, M. Jonker, A. van der Vaart, Amsterdam University Press

Other materials: Lecture slides and exercise materials on elearning.unimib.it

Semester

Second semester

Assessment method

Examination Structure
The exam consists of three parts: written test, oral exam, and an R-based report.

The written test is mandatory, carries a maximum grade of 28, and is passed only with a score of at least 16.

The oral exam is optional and contributes between –1 and +5 points to the final grade.

The report is optional, contributes between 0 and 1 point, must be submitted no later than the day of the oral exam, and is considered only if the sum of the written and oral grades is at least 18.

The final grade is the sum of the scores from the three components.

Written Test
The written test includes:

Open-ended questions on the theoretical concepts covered in the course, including definitions and theorems (with

statements and proofs).

Exercises requiring the application of theoretical concepts and techniques, similar to those discussed in lectures and homework.

During the written test, students are allowed to consult course materials except for a formula sheet, which will be provided on Moodle.

Oral Exam
The oral exam may be requested by either the instructor or the student. It consists of a discussion on the written test, the course content, and the report (if submitted). The same evaluation criteria used in the written test apply to the oral exam.

Report
The report involves the application of the concepts taught in class to an experimental dataset, using the R software.

Office hours

By appointment

Sustainable Development Goals

QUALITY EDUCATION