UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Algebra II

2526-2-E3501Q010

Learning objectives

1. *Knowledge and understanding*
   The student will acquire fundamental knowledge of Algebra, with a particular focus on:

- field extensions, with special focus on finite fields;
- fundamentals of ring theory;
- the theory of modules over a principal ideal domain (PID).
  These topics broaden and consolidate the foundations laid in the Algebra I course, and provide a solid theoretical and conceptual base for more advanced mathematical studies.

2. *Applying knowledge and understanding*
   The student will be able to apply theoretical knowledge to concrete problems, developing technical and operational skills such as:

- determining minimal polynomials of algebraic numbers;
- recognizing and analyzing morphisms between algebraic structures such as groups and rings;
- applying the theory of modules to concrete problems such as the normal forms of matrices.

3. *Making judgements*
   The course aims to foster independent and critical thinking, especially in the context of abstract concepts, through:

- identifying the logical structure of mathematical problems;
- choosing appropriate algebraic tools to solve them;
- interpreting and validating results with rigor.

4. *Communication skills*
   Students will be encouraged to clearly articulate complex mathematical arguments using:

- formal mathematical language;
- logical and coherent exposition;
- the ability to explain concepts to both specialist and, when needed, non-specialist audiences.

5. *Learning skills*
   The course will strengthen the student's ability to learn independently and continuously by providing:

- theoretical and methodological tools for reading advanced texts;
- skills necessary to approach more specialized courses in analysis and other areas of mathematics;
- familiarity with formalization and abstraction, essential for mathematical reasoning and research.

Contents

Fields, rings and modules

Detailed program

FIELDS
Field extensions: algebraic and transcendental extensions, degree of an extension, the degree formula.
Splitting field of a polynomial.
Finite fields: construction, subfields, automorphisms, ciclicity of their multiplicative group.

RINGS
Complements of ring theory.
The Chinese remainder theorem (for polynomials, for commutative rings).
Partial fraction decomposition of rational functions.
Unique factorization domains and Gauss's lemma.
Localizations of a domain. Local rings.
The ring of formal series with coefficients in a field, with some applications.

MODULES
Modules over a ring and linear algebra. Free modules: bases, rank, universal property. Torsion.
Modules over principal ideal domains: finitely generated modules; equivalence of matrices and reduction to normal form.
Structure theorem for finitely generated modules.
Torsion modules and primary decomposition.
Invariant factors and elementary divisors.
Application to abelian groups: structure theorem for finitely generated abelian groups.
Application to canonical forms for matrices: companion matrix, rational canonical form, Jordan canonical form.

Prerequisites

The contents of the courses Linear Algebra and Geometry, and Algebra I.

Teaching methods

This course will be normally be taught entirely by live lectures at the blackboard (DE), which will also video-recorded and made available to the students through the elearning platform.
The precise subdivision is as follows: live lessons at the blackboard (6 CFU); live exercise sessions at the blackboard (2 CFU).
The course is taught in Italian.

Assessment methods

Written exam, followed by oral exam.

The written exam will comprise open questions (not multiple-choice questions) on the theory, and exercises.

The oral examination will be on the theory presented in the lectures, but may include exercises, and possible reference to the text of the written exam.

In both cases the questions will concern definitions, examples, counterexamples, exposition and application of theorems as well as their proofs.

Textbooks and Reading Materials

A PDF of typewritten Notes for the course is made available. The main source for the material is the classical textbook

N. Jacobson, Basic Algebra I, Freeman Co, 1985,

which can be used for further learning.

Semester

First semester

Teaching language

Italian

Sustainable Development Goals

QUALITY EDUCATION