UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Dynamical Systems and Classical Mechanics

2526-2-E3501Q012

Aims

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This course aims to present the basic ideas of Classical mechanics, from the Galielo-Newton formulation to those of Lagrange, Hamilton and Jacobi and to provide the necessary mathematical tools.

The expected learning outcomes include:

1. Knowledge and understanding.

- Knowledge and understanding of the definitions and fundamental statements of the different formulations of Classical Mechanics.
- Knowledge and understanding of some key examples (isolated systems, harmonic oscillator, Kepler's problem, Lagrange's spinning top);

2. Ability to apply knowledge and understanding. The ability to apply the abstract knowledge acquired to the resolution of exercises. In particular, the ability to deduce the Lagrange/Hamilton equations of a constrained system, the ability to reduce the degrees of freedom in the presence of symmetries and the ability in some simple cases to qualitatively discuss the behavior of the solutions of the equations of motion and/or to reduce their solution to quadratures.
3. Autonomy of judgment. The teaching aims to develop the student's ability to critically analyze statements and demonstrations; recognize the validity of a mathematical argument and its interpretation in the specific model studied; independently select appropriate resolution methods depending on the problem addressed. These skills will also be developed through the discussion of multiple resolution methods for the same problem and reflection on the choice of alternative paths.
   (4) Communication skills: students will be able to discuss mathematical descriptions of dynamic system models in a clear and rigorous way, both orally and in writing, and present a demonstration in a coherent and comprehensible way. The use of formal mathematical language is promoted, but also the ability to translate concepts into terms of common language when this is possible.

(5) Learning ability: the teaching aims to provide students with the tools to independently continue the study of the dynamics of classical systems at more advanced levels, to approach new topics with method and rigor, making use of previous knowledge, to use different sources (textbooks, notes, articles) to deepen and update their skills. The teaching, also thanks to its strongly interdisciplinary character in which analytical and geometric tools are used to address the study of abstract and concrete dynamic systems, contributes to building a solid theoretical basis on which to base the path of the degree in mathematics.

Contents

Newtonian Mechanics (a reminder). Ordinary differential equations. Qualitative analysis. The D'Alembert principle and Lagrangian Mechanics. The two-body problem. The rigid body. Hamiltonian mechanics. Canonical transformations.

Detailed program

1. A reminder of the theory of ordinary differential equations. Vector fields and systems of first order ODEs. Equilibria and their stability. Lyapounov theorem. Linearization near equilibrium points. Systems with one degrees of freedom: level curves of the energy

2. Lagrangian Mechanics. Euler-Lagrange equations. Particle constrained on a regular curve. Particle constrained on a regular surface. The D'Alembert principle for general holonomic constraints. Equilibrium points and small oscillations. Variational formulation of Euler-Lagrange equations. One-parameter group of diffeomorphisms, symmetries and Noether's theorem. The two-body problem and the Kepler laws.

3. SO(3) and angular velocity. Inertial and non inertial frames. Mechanics of rigid bodies. The inertia operator. K•oenig's theorem. Euler's equations for rigid bodies. The Euler angles and the Lagrange top.

4. Hamiltonian Mechanics. Legendre transformation. Hamilton's equations. Poisson bracket and Lie bracket. Symmetries and conservation laws in Hamiltonian Mechanics. Variational formulation of Hamilton's equations. Canonical transformations. Liouville's theorem.

Prerequisites

Analysis I, Linear Algebra and Geometry, Physics I.

Teaching form

64 hours of in-person, lecture-based teaching (8 ECTS)
48 hours of in-person, lecture-based exercises classes (4 ECTS)

This course will primarily use lecture-based teaching to deliver the fundamental concepts of Dynamical Systems and Classical Mechanics. The active interaction between teacher and students will take place through questions, discussions and answers during the lessons and exercise classes.

Textbook and teaching resource

The following books are recommended:

1. V. I. Arnold, Metodi matematici della meccanica classica, Editori Riuniti.

2. A. Fasano e S. Marmi Meccanica Analitica Bollati-Boringhieri 2002.

3. L.D. Landau. E. M. Lifshits, Meccanica, Editori Riuniti.

4. N.M.J. Woodhouse, Introduction to analytical dynamics, Oxford Science Publications. The Clarendon Press, Oxford University Press, New York, 1987.

G.Dell'Antonio, Elementi di Meccanica, Liguori, Napoli (1996)

Lecture notes on special parts will be also provided by the teacher.

Suitable collections of solved exercises are:

1. F. Talamucci, Esercizi svolti sul formalismo lagrangiano e hamiltoniano con brevi richiami di teoria. Edizioni Nuova Cultura, 2014

2. Alessandra Celletti, Esercizi e Complementi di Meccanica Razionale, Aracne Editrice, (2003)

3. Giancarlo Benettin, Eserciziario per il corso di Fisica Matematica, Padova (2017)

(freely downloadable from the webpage of the author:

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Semester

Second semester.

Assessment method

The exam consists of two parts: a written test and an oral test.

The written test requires the solution of 2 problems (a problem of Lagrangian Mechanics and a problem of Hamiltonian Mechanis). The duration is typically three hours. Correct answers without clear explanation will not receive full marks. The minimum grade to pass to the oral part is 15/30.

During the oral exam the students will be asked to state and prove the theorems carried out in class and to illustrate their meaning with significant examples. The oral exam will evaluate the knowledge of the theoretical aspects of the course, as well as the ability to expose it in a well-organized and consistent way. When the written test is performed with due theoretical awarness (grade greater or equal to 18/30 and convincing exposure), the teacher could allow the student to renounce to the oral test, with however the upper cutoff of 24/30 in the final grade.

The written an the oral exams equally contribute to the final grade. The oral examination should be taken immediately after the the written test.

During the course, two written partial tests will be offered, each referred to one half of the course. To pass the written examination through the partial tests, the student needs to pass each of them with the minimum grade of 15/30. In this case oral examination must be taken within the exam session of July.

Graduation of marks.

18-19: Scarce knowledge, limited to a reduced part of the program.
Exposition and language competence sometimes incorrect, limited ability of critical elaboration of the material.
20-23: Preparation on a limited part of the course with practical ability but limited autonomus elaboration of the material. Language essentially correct but exposition sometimes unclear.
24-27: Preparation on the majoity of the topics of the course with autonomous argumentation and critical analysis. Satisfying ability in the application of the concepts and techniques and correct and clear exposition and use of language.
28 – 30/30L: Preparation complete on the topics of the course. Autonomous ability of treatment and development of the subjects, and of critical analysis of the problems proposed. Ability in establishing connections between different topics, complete control of the language and of the exposition also in case of complex problems and questions.

Office hours

By appointment through webex platform or in person if possible.

Sustainable Development Goals

QUALITY EDUCATION