UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Measure Theory

2526-2-E3501Q053

Aims

Course Objectives – Measure Theory

Knowledge and understanding
The course provides a solid introduction to the fundamental concepts of measure and integration theory, such as measures, ?-algebras, Lebesgue measure, measurable functions, and integrals. Students will gain a rigorous understanding of these tools, which are essential for advanced studies in mathematical analysis and probability theory.

Applying knowledge and understanding
Students will be able to apply the acquired concepts to solve concrete problems in measure and integration, demonstrating mathematical rigor in handling fundamental results and computations involving the Lebesgue integral, also in elementary contexts of probability and functional analysis.

Making judgements
The course aims to develop students' ability to critically analyze definitions and theorems, assess the logical correctness of proofs, and select appropriate theoretical tools to address problems in broader mathematical contexts.

Communication skills
Students will be able to clearly and rigorously present the theoretical content of the course, using correct mathematical language, both orally and in writing, particularly in the exposition of proofs and discussion of examples.

Learning skills
The course fosters the development of autonomous learning abilities through individual study and problem solving, laying the groundwork for future courses in real analysis, probability, mathematical statistics, and applied mathematics.

Contents

?-algebras and measures. Lebesgue measure. Measurable functions and integration. Lebesgue integral. Modes of convergence. Product measures and multiple integration.

Detailed program

Measure Theory.
Algebras, ?-algebras. Measures, outer measures, Carathéodory's Theorem. Lebesgue measure and Borel measures on the real line.

Integration.
Measurable functions. Integration of nonnegative functions. Monotone Convergence Theorem, Fatou's Lemma. Integration of real-valued functions, Dominated Convergence Theorem. Modes of convergence for sequences of measurable real functions: convergence in L^1, almost everywhere convergence, almost uniform convergence, convergence in measure, Egorov's Theorem. Product measures, Fubini-Tonelli Theorem. n-dimensional Lebesgue integral. Change of variables.

Prerequisites

The contents of Calculus I and II, along with basic knowledge of linear algebra and general topology.

Teaching form

24 hours of lectures delivered in a traditional, in-person format (3 ECTS credits).
12 hours of in-person exercise sessions (1 ECTS credit).

Course taught in Italian.

Textbook and teaching resource

Textbook: G. B. Folland, Real Analysis, Wiley.

Suggested readings:

- L. Ambrosio, G. Da Prato, A. Mennucci: Introduction to Measure Theory and Integration, Edizioni della Normale.
- T. Tao: An introduction to measure theory, American Mathematical Society.

Semester

Spring semester.

Assessment method

The exam consists of a written test, aimed at verifying the level of knowledge, the ability to apply it to the resolution of exercises, the student's independence in making judgements, as well as his/her communication skills. The test is divided into two parts: the first part contains theoretical questions (statements, proofs, definitions, examples/counterexamples illustrated during the course), while the second part contains exercises. The two parts will contribute equally to the determination of the final grade.

Office hours

Upon appointment.

Sustainable Development Goals