UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

# COURSE SYLLABUS

## Calculus and Statistics

**2526-2-E3201Q116**

### Aims

**Knowledge and understanding**
The course provides a solid understanding of the fundamentals of linear algebra, multivariable differential calculus, ordinary differential equations, and descriptive and inferential statistics, with an emphasis on their application to environmental data analysis.

**Applying knowledge and understanding**
Students will develop the ability to apply mathematical and statistical tools to real-world environmental problems, learning to model and interpret phenomena using quantitative methods.

**Making judgements**
The course encourages students to develop independent critical thinking in selecting and justifying appropriate mathematical and statistical approaches to problem solving in the environmental sciences.

**Communication skills**
Students will be able to clearly, rigorously, and effectively communicate both the theoretical knowledge acquired and the results of problem-solving activities, in written and oral form.

**Learning skills**
Upon completion, students will have acquired the prerequisites needed to successfully approach subsequent courses with a modeling or quantitative focus within the Bachelor's degree in Environmental Sciences and Technologies.

### Contents

- Vectors in R? , matrices.

- Differentiation in Rn.
- Differential equations.
- Descriptive statistics.
- Basic probability.
- Inferential statistics.

## Detailed program

- Linear algebra

Vector spaces: sum of vectors, product for a scalar. the vector space R?: inner product, norm of a vector and its properties. Schwarz's inequality, triangle inequality, linear combinations, dependent and independent vectors. Matrices and matrix operations: matrix transpose, matrix addition, product for a scalar and product between matrices. Systems of linear equations and Gaussian elimination method.

- Curves

Vector functions of a real variable, limits, and continuity. Curves, closed curves, simple curves, and planar curves. Support of a curve. Derivative and tangent vector to a curve. Regular and piecewise regular curves.

- Differential calculus for functions of several real variables

Sets in R?. Spherical neighborhoods. Functions of several real variables: introduction and first examples, state functions of thermodinamics. Graphs and level sets. Definition and properties of limits for functions of several variables. Finite limits. Continuous functions. Partial derivatives and gradient, definition of differentiability, link between differentiability and continuity and between differentiability and derivability. Derivability along a given direction and the gradient formula, geometric meaning of the gradient. Sufficient condition for differentiability and the class $C^1$(R?, R). The first differential. Derivative of the composite function: the case p(x) = g(f(x)) with f:R? -> R and g: R -> R and the case p(t) = f(r(t)) with f: R? -> R and r: R -> R?. Level curves and the gradient. Positively homogeneous functions and Euler's theorem, application to thermodynamic potentials. Higher order derivatives and the Hessian matrix. Schwarz's theorem and $C^2$ class. Maxwell's relations in thermodynamics. Vector functions of several real variables, Jacobian matrix. General case of the chain rule. Extremal points. Free and constrained extrema. Stationary points (or critical points). Necessary condition for free extrema (Fermat's theorem). Sufficient condition for the two variables case.

- Differential equations

Definition. Ordinary differential equations and partial differential equations with examples. Exponential growth model and logistic model. Order of a differential equation and systems of differential equations. Differential equations in normal form and equivalence with first-order systems. Cauchy's problem. Cauchy's problem for differential equations in normal form of order n. Existence theorem (Peano) and local existence and uniqueness theorem. Differential equations with separable variables and linear differential equations of the first order. Homogeneous and nonhomogeneous linear differential equations of order n. Structure of the general integral of homogeneous and nonhomogeneous equations. Solution of homogeneous linear equations with constant coefficients of order two. Differential equations associated with RLC circuits and damped harmonic oscillator and their general integrals. Solution of a nonhomogeneous linear equation with constant coefficients when the nonhomogeneous term is a polynomial or an exponential (similarity method). Short overview of the qualitative solution of autonomous differential equations: single equations and 2X2 systems. Qualitative analysis of solutions of the following models: logistic equation; logistic equation with extinction and harvesting; Lotka-Volterra predator-prey model; model for two species in competition.

- Descriptive Statistics

Introduction to statistics: absolute and relative frequency tables, histograms, sample mean and median.
Sample mode, sample variance, sample standard deviation, quartiles, interquartile range, box?plot.
Bivariate data: scatterplot, sample covariance and linear correlation coefficient; correlation does not imply causation.

- Introduction to Probability

Basic concepts: sample space, events, probability measure, definition of probability, fundamental properties of probability.
Independence of events. Introduction to random variables: discrete random variables and their probability mass function; cumulative distribution function of a random variable, the discrete case.
Examples of discrete random variables: uniform, Bernoulli, binomial, Poisson.
Continuous random variables: probability density function (PDF) and cumulative distribution function; properties of the CDF.
Examples of continuous random variables: uniform, exponential.
Expected value of a random variable (discrete and continuous): calculation of the expectation for a Bernoulli and a continuous uniform distribution; properties of the expected value.
Transformations of random variables: illustrative examples; the “law of the unconscious statistician” for computing the expectation of a function of a random variable, with examples.
Variance of a random variable; properties of variance. Independent random variables and the variance of their sum.
Special distributions: normal random variables, standard normal, and chi?square.

- Inferential Statistics

Properties of normal random variables; quantiles of the standard normal distribution and the standard normal table.
Symmetry properties of standard normal quantiles and application to the sample mean of several independent standard normal random variables with the same mean and variance.
Confidence intervals for the unknown mean of a normal population with known variance: two?sided and one?sided intervals at any confidence level.
Central Limit Theorem and its application to confidence intervals for the parameter of a Bernoulli population.
Introduction to hypothesis testing and parametric tests: null and alternative hypotheses; critical region; Type I and Type II errors; significance level; two?sided Z?test for the mean of a normal population with known variance; p?value.
One?sided Z?test for the mean of a normal population with known variance; Z?tests (two?sided and one?sided) for a proportion; Student’s t?test.
Simple linear regression.

## Prerequisites

Differential and integral calculus for real functions of a single real variable. Even if it is not formally required, it is necessary to know and to be able to handle the contents of Mathematics I in order to be able to follow the course profitably.

## Teaching form

24 lessons of 2 hours each of in-person, lecture-based teaching (6 ECTS)
10 exercise classes of 2 hours each of in-person, lecture-based teaching (2 ECTS)

Course delivered in Italian

## Textbook and teaching resource

- Matematica Generale, A. Guerraggio, Bollati Boringhieri. (Linear algebra)
- Analisi Matematica II, M. Bramanti, C.D. Pagani, S. Salsa, ZANICHELLI. (Differential calculus for functions of several real variables and differential equations)
- Probabilità e statistica per l'ingegneria e le scienze, S. M. Ross, Apogeo.
- Esercitazioni di Analisi Matematica 2, M. Bramanti, Esculapio, Bologna. (Exercises)
- Esercizi di calcolo delle probabilità e statistica, D. Bertacchi, M. Bramanti, G. Guerra, Esculapio.
- Probabilità & Statistica 600 esercizi d'esame risolti, M. Verri, Esculapio.

## Semester

First semester

## Assessment method

The exam consists of a mandatory written test and an optional oral test (upon request of either the student or the instructor), which can be taken if a score of at least 18 is achieved in the written test.

The written test is divided into two parts. The first part consists of 4 open questions, each worth 3 points. In this first part, students will be evaluated on their understanding of the fundamental mathematical and statistical concepts covered during the course. Students need to demonstrate their theoretical knowledge and the ability to explain the key conceptscovered in the course. In the second part, students are required to solve some exercises/problems, usually 4. Each exercise is worth 5 points unless otherwise indicated. In this part of the test, the ability to apply the learned knowledge to solve exercises and problems is evaluated, as well as the ability to communicate the solution process clearly and effectively.
The test must be completed within 120 minutes. To pass the exam, a minimum score of 18 is required.

The optional oral test consists of an interview on the topics covered in the course. Both theoretical understanding and the ability to apply mathematical and statistical concepts to solve concrete problems will be assessed. Based on the performance in the oral test, the score obtained in the written test can be increased by a maximum of 4 points or decreased to a failing grade in case of significant unpreparedness.

Partial exams: During the lecture period, there are typically two partial exams that replace the written test if passed. The partial exams follow the same structure as the written test. The first partial exam covers the program taught until the time of the exam, while the second partial exam covers the remaining program. The partial exams are considered passed if a score of at least 18 is obtained in both exams, and the final grade will be the arithmetic mean of the two scores.

During the exams, consulting educational material (textbooks, exercise books, personal notes, formulas) is not allowed, and the use of cell phones, tablets, PCs, smartwatches, etc. is prohibited. The use of a non-programmable and non-graphing scientific calculator is allowed.

## Office hours

By appointment

## Sustainable Development Goals

QUALITY EDUCATION