UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Classical Mechanics

2526-2-E3001Q003

Aims

The content of the Course presents the basic ideas of Classical mechanics, from the Galileo-Newton formulation to those of Lagrange, Hamilton and Jacobi. The necessary mathematical tools for a proper comprehenson of these fundamental theories will be introduced and discussed.

The student, by the end of the course, will have acquired the following skills:

1. Knowledge and understanding
   The student will have acquired a solid but foundational understanding of the Lagrangian and Hamiltonian formalisms of classical mechanics. They will be able to recognize the conceptual meaning of symmetries, constraints, and conservation laws, and understand the role of these tools in the theoretical formulation of physics.

2. Applying knowledge and understanding
   The student will be able to apply the methods learned to analyze and solve mechanical problems, both in simple systems and in the presence of constraints. They will also be capable of reformulating a physical problem using the tools of Analytical Mechanics and applying appropriate mathematical techniques.

3. Making judgements
   The student will be able to critically assess methodological choices in solving a physical problem and identify the most appropriate formalism for modeling it, while distinguishing between various simplifying assumptions.

4. Communication skills
   The student will be able to clearly and rigorously present the main concepts of Analytical Mechanics, using the correct language of physics and mathematics. They will be able to structure and explain the solution to a problem, justifying each step taken.

5. Learning skills

The course will provide the student with the conceptual and methodological foundations needed to successfully approach more advanced courses in theoretical physics and applied mathematics, as well as to independently deepen their understanding of more complex topics in classical mechanics.

Contents

Newtonian Mechanics (a reminder).

Second order differential equations. Qualitative analysis.

Lagrangian Mechanics.

Hamiltonian mechanics.

Detailed program

1) Space-time and events. Newton's principia and the dynamics of point masses.

2) Dynamical systems as mathematical models for physical phenomena. Basic aspects of the theory of second order Ordinary Differential Equations. Phase diagrams of conservative Newtonian systems in one dimension. The Lotka-Volterra system and Volterra's laws. Compartmental models in epidemiology: the SIR model.

Bifurcation diagrams. Linearization of a dynamical system around an equilibrium point. Stability and the theorems of Lyapunov (statement).

3) Dynamics of systems of point masses.

4) Constraints, degrees of freedom, and free coordinates. The D'Alembert principle and Lagrangian Mechanics.

5) The Lagrangian and the Euler-Lagrange equations. Variational principles. Central motions and the Kepler problem. Lagrangian formulation of the Lorentz force. Theory of small oscillations. Further applications. Noether's theorem. Basic notions of the theory of rigid bodies. Applications: rigid bodies in the plane. The Lagrange top.

6) Hamiltonian Mechanics: Hamilton equations and their variatiional formulation. Canonical transformations. Canonical contact (point) transformations. Poisson brackets and constants of the motion. Infinitesimal canonical transformations and Noether's theorem in Hamiltonian Mechanics.

7) Liouville theorem on the conservation of volume in phase space. The Hamilton-Jacobi equation. Complete integrals. Introduction to the notion of separation of variables

Prerequisites

The content of the courses of Calculus I, Linear Algebra and Geometry, Physics I.

Teaching form

- Lectures (5 CFU) via expository teaching. Students will attend lectures where the instructor will present theoretical material and demonstrate problem-solving techniques.

- Classes (3 CFU) via mixed expository and interactive teaching, In expository sessions, students will attend expository-tipe classes where the instructor will apply the theoretical apparatus exposed in the lectures to solve problems in classical mechanics. Besides these expository classes, the course will incorporate interactive teaching methods. This will involve group activities, discussions, and hands-on problem-solving sessions to enhance understanding and foster active participation. We expect that some 10 of classes will be delivered in the interactive way.

Videorecordings of lectures and classes will be available online.

Textbook and teaching resource

References:

L.D. Landau, E.M. Lifshits, “Course of Theoretical Physics, Vol. I: Mechanics” (Pergamon)

H Goldstein, C. Poole, J. Safko, “Classical Mechanics”.

Lecture Notes available on the e-learning page.

Notes of (some of) the lectures, available on the e-learning page.

Semester

First semester

Assessment method

Written and oral examination. The written examination consists in the solution of significant problems in Dynamical Systems, Lagrangian Mechanics and Hamiltonian Mechanics.

Two partial written examinations concerning parts of the program will be held. The first one will be by the end of October, the second one by the end of November/beginning of December. The written part of the examination must be completed in one of the final examination sessions within the July one. After that, students will be asked to solve the full set of problems. Should a student be absent at one of the ongoing partial examination, or in case of fail, he/she will be allowed to solve the corresponding problem in the occasion of his/her first final examination session (still within July).
The first partial written examination will propose a problem in the qualitative theory of dynamical systems in the plane. The second one will concern Lagrangian Mechanics. The written part of the eamination will be completed, in the sessions after the end of the lectures, by a problem in Hamiltonian Mechanics.

The oral examination consists in the discussion of the written part, as well as the discussion of fundamental topics of the course. Questions will be chosen (by the instructor) from a list to be given to the students at the end of the lectures. In the oral session, the student can be asked to solve problems similar to those presented during the course.

The aim of the written examination is basically to assess the achievemnt of points 1) 2), and 3) of the "Aims" Section. Its weight in terms of the final score, is 2/3.

The oral part mainly deals with points 1 and 4) of the "Aims" section.

The examination can be taken in English language. To this end, the interested student must send an email to the instructors at least one week before the written examination.

Office hours

Meetings with individual students or small groups thereof are to be agreed via e-mail or the e-learning page.

Sustainable Development Goals

QUALITY EDUCATION