UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Derivatives

2526-1-F1602M002

Learning objectives

Derivative instruments are financial instruments whose value depends on the value of another financial instrument, called the underlying. This apparently abstract definition covers a number of well-known and easy-to-understand examples such as forward contracts, futures, call and put options on underlyings such as stocks, bonds, indices, commodities, volatility, electricity, and many others.

Derivatives have sometimes an unjustified reputation of being complex or excessively risky. On the contrary, as we will see,
derivatives have many essential functions in modern financial markets such as hedging and transfering risk, conveying information about the underlying, allowing a broad range of trading strategies, making the volatility herself a marketed asset.
The aim of this course is to provide the basic knowledge about these instruments and the mathematical and statistical tools involved in their use that is essential for any student of Finance.

Some specific course objectives are the following:

1) Knowing the basic types of derivative instruments and understanding their uses and financial significance

2) Understanding the concept of a mathematical model of a financial market and its use in the valuation of a derivative instrument

3) Knowing all details of the models studied and understanding the derivations of all results

4) Being able to apply the studied models to the pricing or hedging of a generic derivative instrument

5) Knowing the most important functions of the Bloomberg terminal related to the visualization and the analysis of financial derivatievs, such as OMON, OVME, SKEW.

Contents

- Basic concepts on options

- Multi-period binomial model

- One-period models

- Continuous-time models

- Black-Scholes model

- Merton model

- Vasicek model

- The Monte Carlo method and delta-hedging

- The VIX Index

- The main Bloomberg functions on options: OMON, OV, OSA, SKEW.

Detailed program

Basic Concepts on Options
Recalls on derivative instruments: forward contracts, futures contracts, call and put options and examples of applications. Payoff and replication concept; forward price and spot-forward parity. Put-call parity. Combinations of options (spread, butterfly, strangle, straddle). Convexity of the call price as a function of the strike. Super-replication and sub-replication. Merton constraints. American options and optimality of early exercise. Qualitative discussion of factors influencing option prices.

Multi-period binomial model
The one-period binomial model: derivation of the formula for the price of a generic payoff.
The biperiodal binomial model and its use for the valuation of American options.
The multiperiod binomial model: formula for valuing a generic payoff and derivation of the formula for the European call. Choice of u and d parameters and historical volatility.

One-period models
One-period models with an arbitrary number of securities and world states. Payoff matrix.
Replicability, market completeness, characterisation of completeness.
Definition of arbitrage opportunities. Definition of the vector of state prices and first fundamental theorem of valuation. Second fundamental theorem of valuation.
Superreplication and subreplication as a linear programming problem.

Models in continuous time
Definition and first properties of Brownian motion
Ito processes: definition and examples (Brownian motion with drift, geometric Brownian motion)
Ito formula: drift and volatility of a transformed process
Geometric Brownian motion, recalls on lognormal distribution.

The Black-Scholes model
Model assumptions. Derivation of the Black - Scholes differential equation. Special solutions, superposition

principle. Derivation of the BS formula as the expected discounted value of the payoff. First properties of the BS formula. Parameter dependence and calculation of greeks. Approximation for short-term ATMF options. First extensions of the BS model: presence of induced flows. Empirical verifications of the BS model. The implied volatility and the smile.

The Merton model
Generalities on credit risk. The Merton model. Calculation of the risk-neutral probability of default. Analytical derivation of the spread rate curve.

The Monte Carlo method
Simulation of random numbers. Calculation of the price of a derivative instrument using the Monte Carlo method. Delta-hedging.

The VIX Index.

The main Bloomberg functions on options: OMON, OV, OSA, SKEW.

Prerequisites

Some of the basic knowledge of Mathematics, Financial Mathematics and Statistics required for admission to the Master's degree in Economics and Finance. In particular:
- Mathematics: elementary functions, limits, derivatives, integrals, convexity and concavity, vectors, matrices, operations between matrices, linear combinations, linear dependence and independence, rank of a matrix, determinant, inverse matrix, solution of linear systems.
- Financial mathematics: capitalisation and discounting, duration, basic concepts on options (which will however be refreshed at the beginning of the course)
- Statistics: mean, variance, covariance, correlation and their properties. Fundamentals of
probability: sample space, events, probability, random variables. Discrete and continuous random variables, in particular the binomial, the normal, the lognormal and their properties.

Teaching methods

Classes are held in person. Teaching materials are made available to students before the lesson. If a student is unable to attend a class, they can study the materials independently, and if anything is unclear, they can ask questions in the dedicated forums on the e-learning page. Each question receives a response from the professor.

Interactive Teaching
Classes are conducted in two-hour blocks. In each lesson, a variable amount of time between 15 and 45 minutes is dedicated to interactive activities such as:
i) discussion of problems assigned as homework in previous lessons, whose solutions form an integral part of the course
ii) implementations in Excel or Matlab Online using a bring your own device approach
iii) questions posed to students individually or in groups
iv) collective critical use of AI tools such as ChatGPT

Assessment methods

Within the first two weeks, students will have to choose between two alternative possible exam formats: *frequentante* (attending) or *non frequentante* (not attending).
Students choosing the *non frequentante* format will have to pass an exam at the end of the course consisiting in a written and in a oral part. Roughly, the written exam will be mainly aimed at testing the ability of solving exercises, while the oral exam will investigate the understanding and the ability to express the theoretical concepts.
Students choosing the *frequentante* format will be required to actively participate in a number of interactive activities during the lectures (problem-solving sessions, timely completion and submission of assignments, activities at the Bloomberg Terminal, formative assessment quiz, and so on). They will have the additional possibility of splitting the written exam in two with a *prova intermedia* (midterm exam) and will not have an oral part.
Students not expressing their preference or students choosing the *frequentante* format but not completing the required activities will be moved to the *non frequentante* group.

Textbooks and Reading Materials

- Lecture notes and materials provided by the lecturer.

For further reading:
- J. Hull "Option and futures"
- J. Cox, M. Rubinstein "Option markets"

Semester

Second semester.

Teaching language

English.

Sustainable Development Goals