UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Algebra I

2526-1-E3502Q001

Aims

1. Knowledge and understanding
   At the end of the course, students will have acquired a solid understanding of the fundamental algebraic structures (groups, rings, fields) and the core methods of abstract algebra. They will be able to recognize, describe, and compare these structures, understanding their properties and interrelations.

2. Applying knowledge and understanding
   Students will be able to apply the theoretical concepts learned to solve algebraic problems, both through routine exercises and in more complex situations requiring autonomous reasoning. They will be capable of using algebraic methods to formalize and address problems in mathematical and, where appropriate, interdisciplinary contexts.

3. Making judgements
   Students will develop the ability to critically analyze algebraic problems, evaluate different solution strategies, and choose the most appropriate tools. They will be able to formulate conjectures and support their validity through rigorous logical reasoning.

4. Communication skills
   Students will be able to clearly and rigorously express concepts, definitions, theorems, and proofs, using the formal language of algebra. They will also be able to discuss solutions and strategies with peers and instructors, both orally and in writing.

5. Learning skills
   The course will equip students with the conceptual and methodological tools necessary to pursue further studies in algebra and related mathematical areas independently. Students will be encouraged to adopt an active approach to learning, through problem-solving, consulting advanced texts, and reflecting on the meaning and application of algebraic concepts.

Contents

Sets, Relations, Operations; Modular Arithmetic; Elements of Group and Ring Theory; Polynomials;

Detailed program

A) Sets, Relations, operations: Axiom of choice; order relations; equivalence relations; congruences.

B) Arithmetic properties of the set of integers Z; modular arithmetics; residue classes.

C) Basics of Group Theory; subgroups, subgroup generated by a subset; cyclic groups; cosets; Lagrange's Theorem; congrunces in a group; normal subgroups; group homomorphisms and quotient groups; main theorems on homomorphisms; automorphisms; direct and semidirect products; symmetric and alternating groups; permutation groups; group actions (G-sets); regular representation; Cayley's Theorem; conjugation action; orbits: examles and applications; Sylow's Theorems.

D) Basics on Ring Theory: domains, division rings, fields; ring homomorphisms: ideals, quotint rings, elementary theory of homomorphisms; Chinese remainder Theorem; divisibility in a domain; embeddings of domains into fields; prime and maximal ideals; principal ideal and unique factorization domains.

E) Polynomial algebras: polynomials in one variable over a field; decomposition into irreducible factors.

Prerequisites

Basic notions of high school algebra and analysis

Teaching form

The course is organized in Lectures (48 hours, 6 CFU) and Exercise classes (24hours, 2CFU). Definitions, results and relevant theorems will be presented in Lectures, providing examples and problems making use of the notions introduced.

Textbook and teaching resource

Textbook
I. N. Herstein, Algebra, Editori Riuniti
Written notes on some topics available on this platform.

Suggested readings:
Jacobson, Basic Algebra I, Dover, 1985

Semester

Second semester

Assessment method

Written and oral examination (18-30/30).

The written examination evaluates the knowledge of the course contents and the ability to apply them to problem solving. The correctness of the answers, the mathematical language as well as the rigor and clarity of the exposition will be evaluated. Two partial tests, during the course, allow you to take the oral test.
The oral examination consists of an interview on the course contents and can only be taken if the written test is sufficient.

During the year there are 6 exam sessions and 2 ongoing tests.

Office hours

By appointment via e-mail.

Sustainable Development Goals

QUALITY EDUCATION | GENDER EQUALITY