UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematical Analysis I

2526-1-E3502Q002

Aims

**Learning Objectives according to the Dublin Descriptors

**Knowledge and understanding

Students will acquire a solid understanding of the fundamental concepts of Mathematical Analysis for real-valued functions of a single variable, developed with logical and rigorous methods. They will understand the theoretical foundations of limits, differential and integral calculus, sequences, and series, in line with the modern tradition of Mathematical Analysis.

**Applying knowledge and understanding

Students will be able to autonomously and flexibly apply the acquired knowledge to solve exercises, mathematical problems, and basic applications, even in contexts beyond pure theory. They will develop appropriate techniques and problem-solving strategies.

**Making judgements

Students will develop the ability to critically analyze the concepts learned and to independently assess the logical consistency and correctness of definitions, theorems, proofs, and problem-solving methods. Personal reflection and the ability to connect different areas of Analysis will be encouraged.

**Communication skills

Students will be able to clearly, rigorously, and effectively communicate theoretical content using proper mathematical and formal language, both in written and oral form. They will also be able to present the solutions of exercises and problems in a well-structured and justified manner, demonstrating mastery of mathematical vocabulary.

**Learning skills

The course will provide students with the conceptual and methodological tools needed to successfully tackle subsequent mathematics courses. Independent study skills will be fostered, together with a learning approach based on deep understanding, logical reasoning, and reflective practice in problem-solving.

Contents

Real and complex numbers. One-variable calculus: limits, continuity, differential calculus, integration. Sequences and series.

Detailed program

Real numbers. Field axioms, order axioms, rational numbers, the completeness axiom. The Archimedean property of the real-number system. Supremum and infimum of a set, properties of the supremum and the infimum. Natural numbers as a subset of **R**. Integer and rational numbers. Sum, product and factorial symbols. Integer part and modulus of a real number. Density of **Q** in **R**. Number *e*.

Complex numbers. Definition, algebraic form, modulus, conjugate of a complex number, real part and imaginary part, triangle inequality. Trigonometric and exponential form of a complex number, products and power of complex numbers in trigonometric/exponential form. Complex exponentials. Roots of complex numbers. Fundamental theorem of algebra.

Functions. Definition, domain, codomain, and range. Injective and surjective functions, bijections. Composition of functions, inverse functions, restriction. Real-valued functions of one real variable, the graph of a function. Monotonic functions, supremum and infimum, maximum and minimum. Elementary functions and their graphs (powers, exponentials, logarithms, trigonometric functions and their inverses, absolute value function, integer part, fractional part, sign function).

Limits. Definitions, examples, properties: uniqueness of the limit, Sign Permanence Theorem, Squeeze Theorem. Limit of sum, product, quotient and composition of functions. Special limits. One-side limits. Limits of monotonic functions. Landau symbols. Comparison of infinitesimals.

Numerical sequences. Limits of sequences. Boundedness of converging sequences. Subsequences. Existence of a convergent subsequence for a bounded sequence. Monotonic sequences. The number *e.* Cauchy sequences. Upper and lower limits.

Continuity. The definition of continuity of a function. Composite functions and continuity. Sign Permanence Theorem. Bolzano's theorem. The intermediate-value theorem. Continuity of the inverse function. Continuity of elementary functions: powers, exponentials, logarithms, trigonometric functions and their inverses. Sequential criterion for the continuity of a function. Weierstrass theorem. Uniform continuity. Heine-Cantor theorem. Discontinuities. Lipschitz continuity.

Series. Definition. Convergent series, divergent series. Telescoping series, geometric series. Necessary condition for convergence of series. Absolute convergence. Series of nonnegative terms: comparison test, root test and ratio test. Alternating series: Leibniz's test.

Differential calculus. The derivative of a function. Geometric interpretation of the derivative as a slope. Left-hand and right-hand derivatives. Continuity of differentiable functions. The algebra of derivatives. The chain rule for differentiating composite functions. Derivatives of inverse functions. Derivatives of elementary functions. Extreme values of functions. Fermat's theorem. Rolle's theorem. The mean-value theorem for derivatives and applications. Relation between monotonicity and sign of the derivative. Cauchy's generalized mean value theorem. De l’Hôpital's rule. Convex and concave functions. The sign of the second derivative and the convexity/concavity of a function. Inflection points. Taylor's formula with Peano form of the remainder. Taylor's formula with mean-value form of the remainder.

Integral calculus. Step functions, definition of the integral for step functions. Properties of the integral of a step function. Upper and lower integrals on bounded intervals. Riemann integral. Properties of the Riemann integral (linearity, monotonicity). Integrability of the positive/negative part and of the modulus of an integrable function. Integrability of the restriction of an integrable function, integral over oriented intervals, additivity with respect to the interval of integration. Integrability of monotonic functions and continuous functions. Mean-value theorems for integrals. Fundamental theorem of calculus. Antiderivatives. Integration by parts, change of variable. Integration of rational functions. Improper integrals.

Prerequisites

Elementary algebra, elementary trigonometry, elementary analytic geometry.

Teaching form

64 hours of in-person, lecture-based teaching (8 ECTS)
48 hours of in-person, lecture-based exercises classes (4 ECTS)

Course delivered in Italian

Textbook and teaching resource

Textbook: E. Giusti, Analisi Matematica I, Bollati Boringhieri.

Suggested readings:

- G. De Marco: Analisi Uno, Zanichelli Decibel.
- C. D. Pagani, S. Salsa: Analisi matematica 1, Zanichelli.

Exercise books:

- E. Giusti: Esercizi e complementi di analisi matematica, volume 1, Bollati Boringhieri.
- G. De Marco, C. Mariconda: Esercizi di calcolo in una variabile, Zanichelli Decibel.
- S. Salsa, A. Squellati: Esercizi di analisi matematica 1, Zanichelli.
- E. Acerbi, L. Modica, S. Spagnolo: Problemi scelti di analisi matematica. Vol. 1, Liguori.

Semester

First year, First semester.

Assessment method

Written and optional oral examination (18-30/30).

The written examination assesses knowledge of the course content and the ability to apply it to problem-solving. It also requires the exposition of statements and proofs of theorems, definitions, examples/counterexamples, and calculation techniques. Evaluation criteria include the correctness of answers, mathematical language proficiency, and the rigor and clarity of the exposition.

An optional oral examination, consisting of an interview on the course content, is available only to students who pass the written test.

During the year there are 6 exam sessions.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION