UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Algebra Lineare e Geometria

2526-1-E3005Q001

Aims

In line with the educational objectives of the Degree in Mathematics, the course aims to provide an introduction to linear algebra with applications to geometry, essential to prepare the student to understand the mathematics that will be taught in other courses.
(1) **Knowledge and understanding**: students will acquire a solid understanding of the fundamental concepts of linear algebra and geometry. In particular, they will become familiar with: vector spaces, diagonalization, bilinear forms and scalar product and geometric structures in Euclidean space. The development of such knowledge is grounded in rigorous theoretical understanding, supported by examples and applications.
(2) **Applying knowledge and understanding**: students will be able to apply the acquired knowledge to
solve algebraic and geometric problems using both theoretical and computational tools and to reproduce the proofs presented in the course. Exercises and problem-solving activities are designed to enhance the ability to recognize and apply algebraic and geometric models.
(3) **Making judgments**: the course aims to develop students' ability to critically analyse mathematical statements and proofs, assess the validity of mathematical arguments and identify logical errors, select independently appropriate solution methods depending on the problem. These skills are encouraged through comparative discussion of different approaches to the same problem and critical reflection on definitions and alternative perspectives.
(4) **Communication skills**: students will be able to clearly and rigorously express mathematical concepts, both orally and in writing, present mathematical proofs in a coherent and comprehensible manner. The use of formal mathematical language will be promoted, alongside the ability to translate ideas into more accessible terms.
(5) **Learning skills**: the course aims to equip students with the ability to continue studying algebra and geometry at a more advanced level, approach new topics methodically and rigorously, building on previously acquired knowledge, use diverse sources (textbooks, lecture notes, academic papers) to deepen and update their understanding. This course provides a foundational theoretical background that supports the entire undergraduate mathematics programme.

Contents

Vector spaces; systems of linear equations; linear maps; matrices; diagonalization of an endomorphism; scalar products; affine and euclidean geometry.

Detailed program

- Systems of linear equations: Gaussian elimination method, Rouchè-Capelli Theorem.
- Matrices: matrix product, rank, the ring of square matrices and invertible matrices.
- Vector spaces: generators, basis and dimension; linear subspaces; Grassmann Theorem.
- Linear maps: kernel and image, relation between rank and nullity, matrices associated to linear maps and isomorphisms.
- Determinant of a square matrix and properties; Laplace theorem and Binet theorem.
- Eigenvalues and eigenvectors of an endomorphism; characteristic polynomial of endomorphisms of finite dimensional vector spaces, diagonalization.
- Dual space and dual base.
- Scalar products, orthogonal basis and Sylvester Theorem; Euclidean spaces and Gram-Schmidt process.
- Self-adjoint operators and spectral Theorem.
- Affine spaces, affine coordinate systems, affine subspaces and their representations. Distance and orthogonality.
- Euclidean classification of plane conics.

Prerequisites

Good knowledge of high school mathematics.

Teaching form

The course is organized as follows:
-Lectures (48 hours equal to 6 ECTF) in person;
-Exercises classes (24 hours equal to 2 ECTF) in person.

Both provide lecture-based teaching to deliver the fundamental concepts of Linear Algebra and Geometry..
Definitions, results, and relevant theorems will be discussed in Lectures, providing examples and problems making use of the notions introduced. Exercises on the subject matters covered in the lectures are presented and solved during Exercise classes.

Some exercise sets will be made available regularly on the e-learning website to encourage participation. At the webpage of the course students can find self-assessment quizzes realating to topics covered in the lectures.

A tutor will provide students with support in solving the exercises published on the e-learning website.

The course is delivered in Italian.

Textbook and teaching resource

Reference books:

- M. Abate, Geometria, McGraw Hill, 2002.
- S. Lang, Algebra Lineare, Boringhieri, III edizione.
- E. Schlesinger, Algebra lineare e geometria, Zanichelli 2017

Lecture notes on the e-learning webpage.

Semester

First semester.

Assessment method

The assessment consists of two parts: a **written exam** and an **oral exam**, both evaluated based on correctness, completeness, use of rigorous mathematical language, and clarity of the answers provided.

There will be **three midterm tests**. The dates of these tests will be scheduled and communicated to students at the beginning of the course. Each test involves independently solving assigned exercises at home, based on the material covered, and submitting the solutions within the specified deadline. Each test is worth 10 points and is evaluated based on accuracy, rigor of the procedure, and the mathematical language used. If the total score from the three tests is at least 18 out of 30, it will be converted into a **bonus** (up to a maximum of 3 points), which contributes to the final grade. This bonus expires after the first two exam sessions.

The **written exam** includes:

- a first part consisting of multiple-choice questions, which must be passed in order for the remaining part to be evaluated;
- a second part consisting of open-ended exercises, similar to those discussed during practice sessions. These aim to assess the student's ability to apply theoretical results to problem-solving and theoretical questions, requiring definitions and theorem statements, the discussion of examples and topics covered during the course, and the reproduction of verifications or simple proofs. The goal is to assess the understanding of fundamental notions and concepts presented during the course.

The maximum score is 32 points: up to 24 for solving exercises, and up to 8 for theoretical questions. The written exam is considered passed with a total score of at least 18 points.

Oral exam. Admission to the oral exam is conditional on passing the written exam. The oral begins with a discussion of the written exam and continues with questions requiring definitions, examples and/or counterexamples of concepts introduced during the course, as well as the statements and proofs of the theorems presented in class. The aim is to assess the student's understanding and mastery of the course material, as well as their ability to rework the concepts learned and present them with rigor.
The exam is considered passed only if the oral is deemed satisfactory.
The grade proposed at the end of the oral exam also takes into account the score of the written exam, increased by any bonus from the midterm tests. This grade constitutes the final exam grade.
The exam is passed with a final grade of 18 or higher.

It is possible to be **exempted from the oral exam**. Students who have passed the written exam have two options:

- take the oral exam;
- register a grade equal to the minimum between *S* and 27, where *S* is the score obtained in the written exam, increased by any applicable bonus.

Note that any bonus earned **does not contribute to passing the written exam**.
Also note that to record a final grade higher than 27, the oral exam **must** be taken.
At the instructors' discretion, an oral exam may be made mandatory if the theoretical part of the written exam reveals serious deficiencies.

There will be **six exam sessions**, and the dates will be published on the course's e-learning page.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION