



UNIVERSITÀ  
DEGLI STUDI DI MILANO-BICOCCA

## COURSE SYLLABUS

### Relativistic Astrophysics

2526-1-F5803Q003

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#### Aims

Application of fundamental concepts of special and general relativity to the field of astrophysics

At the end of the course the students:

- 1- will have a basic knowledge of the fundamental concepts of special and general relativity
- 2- will know the basic solutions of Einstein equations in vacuum (Schwarzschild/Kerr) and with matter (TOV equations) and their essential properties
- 3- will know the basic concepts of gravitational lensing
- 4- will be familiar with the physics of compact objects such as white dwarfs, neutron stars and black holes and will see the foundation of timing of millisecond pulsars to test General Relativity
- 5- will know the basics of the physics of accretion onto compact objects and the dynamics of light and particles in curved spacetimes
- 6- will have a general picture of the formation, evolution and dynamics of supermassive black holes
- 7- will learn basic notions of gravitational waves

#### Contents

- 1- Basics of special and general relativity
- 2- Physics of compact objects
- 3- Orbits in Schwarzschild and Kerr metrics
- 4- Gravitational lensing
- 5- Timing of millisecond pulsars
- 6- Accretion theory
- 7- Formation and evolution of supermassive black holes
- 8- Dynamics of binary systems (stellar and supermassive)
- 9- Gravitational waves from binary systems

## Detailed program

### **I- SUMMARY OF SPECIAL AND GENERAL RELATIVITY** - space time

- Lorentz transformations
- relativistic invariants, mass-energy
- equivalence principle
- symmetries and Killing vectors
- Einstein's field equations

### **II- SOLUTIONS TO EINSTEIN EQUATIONS** - vacuum solutions: Schwarzschild and Kerr metrics

- relativistic equations of motion
- motion of massive and massless particles in the aforementioned metrics
- relativistic potential, light ring and last stable orbit
- light bending and periastron precession
- radial fall in a gravitational field
- Shapiro delay
- solutions with matter and TOV equations
- Schwarzschild solution with matter

### **III- GRAVITATIONAL LENSING** - derivation of light bending from the variational principle

- lensing equation
- point-like lenses and multiple images
- astrophysical applications

### **IV- GRAVITATIONAL WAVE (GW) EMISSION** - linearization of Einstein's

- general GW solution in vacuum (basics)
- GW detection via interferometers
- GW solution for binary systems (basics)
- panorama of astrophysical GW sources

### **V- TIMING OF MILLISECOND PULSARS** - introduction to the pulsar timing concept

- derivation of the formula for the time of arrival of the pulses of a pulsar in a compact binary system
- tests of General Relativity and notable binary systems

### **VI- FORMATION, EVOLUTION AND DYNAMICS OF MASSIVE BLACK HOLES (MBHs)**

#### **1- First barionic structures: formation of 'seed' black holes**

- basics of early cosmic structure evolution
- models of seed black hole formation
- remnant of population III stars
- direct collapse

#### **2- MBH growth along the cosmic history** - introduction to the growth routes of MBHs, accretion theory

- spherical accretion (Bondi)
- disk-like accretion (Shakura-Sunyeu)
- implications for the MBH mass growth and spin evolution
- Soltan argument
- scaling relations between MBHs and host galaxies
- galaxy mergers and MBH binary coalesce

#### **3- Formation and evolution of massive black hole binaries (MBHBs)** - Chandrasekhar's dynamical friction

- practical application to the isothermal sphere
- MBHB orbital shrinking in dense environments
- interaction with stars
- interaction with circumbinary disks

- emission of GWs and final coalescence

## Prerequisites

None, besides the basic classes of the bachelor

## Teaching form

42 hours of frontal lectures, mostly at the blackboard, occasionally with the support of slides (6 credits)

20 hours of exercises and supporting activities (2 credits)

Lectures will be in English.

Recording so the lectures (or equivalent lectures from previous years) will be made available to meet the needs of students who might be impeded in personally attending classes.

## Textbook and teaching resource

**Supporting material will be uploaded on e-learning during the course of the semester, in any case here follows an (incomplete) list of useful references.**

### I- SUMMARY OF SPECIAL AND GENERAL RELATIVITY

A first course of General Relativity, B. Schutz

Notes on General Relativity and gravitational waves from V. Ferrari (will be distributed via e-learning during the course)

### II - RELATIVISTIC OBJECTS IN THE UNIVERSE

Black holes, white dwarfs and neutron stars: the physics of compact objects, S. Shapiro and Teukolsky

### III- RELATIVISTIC DYNAMICS AND THEORY OF ACCREDITATION

Black holes, white dwarfs and neutron stars: the physics of compact objects, S. Shapiro and Teukolsky

J. Frank, A. King, D. Raine, "Accretion power in astrophysics":  
[http://qxyang.lamost.org/uploads/books/Accretion\\_Power\\_in\\_Astrophysics.pdf](http://qxyang.lamost.org/uploads/books/Accretion_Power_in_Astrophysics.pdf)

### IV-MASSIVE BLACK HOLE FORMATION, EVOLUTION AND DYNAMICS

#### 1-Evolution of cosmic structures

Barbara Ryden, "Introduction to cosmology", Chapter 12:  
[http://carina.fcaglp.unlp.edu.ar/extragalactica/Bibliografia/Ryden\\_IntroCosmo.pdf](http://carina.fcaglp.unlp.edu.ar/extragalactica/Bibliografia/Ryden_IntroCosmo.pdf)

Abraham Loeb, "First Light":

Jarle Brinchmann, galaxy formation lectures:

#### 2-First baryonic structures: seed black hole formation

Marta Volonteri, "Formation of supermassive black holes":

Yoshida et al., "Formation of Primordial Stars in a LCDM Universe":

Marta Volonteri & Bernadetta Devecchi, "Formation of the first nuclear clusters and massive black holes at high redshift"

3-MBH growth along the cosmic history

Celoria et al., "Lecture notes on black hole binary astrophysics":

King et al., "Aligning spinning black holes and accretion discs":

J. Binney & S. Tremaine, "Galactic Dynamics", 1987 (dynamical friction, loss cone theory, stellar hardening)

D. Merritt, "Dynamics and Evolution of Galactic Nuclei", 2013 (dynamical friction, loss cone theory, stellar hardening)

4-Formation and dynamical evolution of massive black hole binaries (MBHBs)

Celoria et al., "Lecture notes on black hole binary astrophysics":

J. Binney & S. Tremaine, "Galactic Dynamics", 1987 (dynamical friction, loss cone theory, stellar hardening)

D. Merritt, "Dynamics and Evolution of Galactic Nuclei", 2013 (dynamical friction, loss cone theory, stellar hardening)

V- GRAVITATIONAL WAVES FROM BINARY SYSTEMS

Valeria Ferrari lecture notes (available on e-learning)

Michele Maggiore: "Gravitational Waves". Book 2, 2018

## **Semester**

First semester.

## **Assessment method**

Oral examination. The student will first be asked to elaborate on a topic of his choice for about 15-20 minutes. In the rest of the exam, the lecturer will ask other questions covering any of the topics treated in class.

The exam will evaluate:

- the acquired knowledge of the topics treated during lectures
- the ability to perform analytical derivations
- the ability to critically tackle problems related to the material studied in class

There will be no intermediate examinations nor homework.

## **Office hours**

Any day is possible, so long as an appointment is requested via email. I generally use Google Meet for remote meetings.

## **Sustainable Development Goals**

QUALITY EDUCATION | INDUSTRY, INNOVATION AND INFRASTRUCTURE

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