UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Geometric Methods for Theoretical Physics

2526-1-F1703Q011

Aims

Introduction to Differential and Complex Varieties and Algebraic Topology.

1. Knowledge and understanding
Students will acquire a solid understanding of the fundamental tools of differential geometry and algebraic topology, with particular focus on:

- differentiable structures on smooth manifolds, tensors, differential forms, and de Rham cohomology;
- complex and holomorphic manifolds, with insights into conformal geometry and field structures;
- basic algebraic topology, including homotopy, fundamental groups, and cohomology theory.
  These concepts will be framed within the context of modern theoretical physics (e.g., field theory, quantum mechanics, general relativity).

2. Applying knowledge and understanding
Students will be able to apply the acquired concepts and tools to:

- analyze physical models formulated on manifolds (e.g., space-time in general relativity);
- interpret physical structures in geometric terms (bundles, connections, gauge theories);
- compute geometric and topological invariants relevant to theoretical contexts;
- connect mathematical formalisms (differential, topological, complex) to concrete physical phenomena.

3. Making judgements
The course will foster students' critical thinking in selecting the most appropriate mathematical tools to model complex physical situations. Students will develop autonomy in evaluating the internal consistency of physical theories in geometric and topological terms, and in distinguishing between essential and redundant structures in theoretical models.

4. Communication skills
Students will develop the formal language needed to clearly explain advanced mathematical concepts and to

effectively interact with peers in mathematics and theoretical physics. They will be encouraged to write structured texts and deliver short oral presentations, with attention to rigor and terminological precision.

5. Learning skills
The course will provide a foundation for independent study of geometric and topological methods in physics. Students will be able to approach advanced texts and research articles and continue their studies in specialized or research-oriented courses (e.g., Riemannian geometry, gauge theories, differential topology).

Contents

Differentiable and Riemannian manifolds, differential forms and cohomology, Riemann surfaces and complex manifolds, coverings and fundamental group.

Detailed program

- Theory of Differentiable Manifolds
  Definition and initial properties of differentiable manifolds, differentiable maps, and bundles, differential forms, and de Rham cohomology. Riemannian manifolds (brief introduction).
- Complex Geometry
  Riemann surfaces, holomorphic and meromorphic maps, line bundles. Complex manifolds, complex bundles.
- (Algebraic) Topology
  Covering theory, liftings, homotopy, fundamental group.

Prerequisites

Undergraduate Mathematics Courses.

Teaching form

24 2-hour lectures, delivered in-person in a didactic format. In Italian.

Textbook and teaching resource

Milnor, J. Topology from a differentiable viewpoint
Jost, J. Compact Riemann Surfaces
Huybrechts, D. Complex Geometry: an introduction
Petersen, P. Riemannian Geometry
Hatcher, A. Algebraic Topology

Semester

First semester

Assessment method

Oral exam on the course content, including further insights or solving simple exercises. The grade is comprehensive.

Office hours

By appointment

Sustainable Development Goals

QUALITY EDUCATION | REDUCED INEQUALITIES