UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Mathematical Methods for Physics

2526-1-F1703Q027

Aims

Group and representation theories and their applications to theoretical physics.

Contents

Finite groups, matrix Lie groups, and their representations.

Detailed program

These lectures are organised into four main parts:

1. An introduction to groups
   Definitions of groups and subgroups, permutation groups, cosets, Lagrange's theorem, isomorphisms, the dihedral group, conjugacy classes, normal subgroups and quotient groups, the classification of groups of small order, the first isomorphism theorem, group actions and the orbit-stabiliser theorem, Cayley's theorem.

2. The representation theory of finite groups
   Complete reducibility and Schur’s Lemma, inner products and unitary representations, characters and their properties, tensor products and dual representations, applications to quantum mechanics.

3. Matrix Lie groups, Lie algebras, and their representations
   Matrix Lie groups, Lie algebras, relationships between Lie groups and Lie algebras, the complexification of a real Lie algebra. Examples of the representations of sl(2,C) and sl(3,C): the irreducible representations of

SU(2) and SO(3), the theorem of the highest weight for sl(3,C). General sl(n,C): the Weyl character and dimension formulae, characters, tensor product decompositions and branching rules.

4. The general theory of semisimple Lie algebras
Cartan subalgebras, roots and weight spaces, the Weyl group, root systems and their properties, explicit root systems of the classical Lie algebras, Dynkin diagrams, the Cartan classification of simple Lie algebras.

Prerequisites

Undergraduate degree in maths or physics.

Teaching form

Lessons (6 CFU), This course will be taught in English.

Textbook and teaching resource

We follow closely the following lecture notes: https://bit.ly/mibgrouprep

Finite groups:

1. A. F. Beardon, Algebra and Geometry. Cambridge University Press, 2005.
2. M. Artin, Algebra. Prentice Hall, 1991.
3. T. W. Körner, Groups and geometry. https://www.dpmms.cam.ac.uk/~twk10/Alg.pdf

Representation theory, Lie groups and Lie algebras:

1. W. Fulton and J. Harris, Representation theory : a first course. Graduate Texts in Mathematics, January 1991, Springer New York, NY, 1991.
2. G. James and M. Liebeck, Representations and Characters of Groups. Cambridge University Press, 2 ed., 2001.
3. B. C. Hall, Lie Groups, Lie Algebras, and Representations. Graduate Texts in Mathematics, January 2015, Springer Cham, 2015.

Applications to theoretical physics:

1. L. Landau and E. Lifshitz, Quantum Mechanics: Non-Relativistic Theory. Course of theoretical physics. Butterworth-Heinemann, 1981.
2. A. Zee, Quantum Field Theory in a Nutshell: Second Edition. Princeton University Press, 2, 2010.

Semester

First semester

Assessment method

Oral exam. Open questions on all course's topics covered during the lectures.

Office hours

By appointment, by sending an e-mail to *n.mekareeya@gmail.com*

Sustainable Development Goals

QUALITY EDUCATION