UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Matematica per l'Economia M

2526-1-F8206B025

Learning objectives

The course in Mathematics for Economics M aims to provide students with the fundamental skills for mathematical analysis applied to economic models, developing the ability to understand and apply mathematical techniques in economic contexts. In particular, the educational objectives include the acquisition of knowledge on differential equations and dynamic systems, with particular attention to explicit resolution, qualitative analysis and stability of solutions, as well as the understanding of existence and uniqueness theorems. The course also aims to develop skills in the field of dynamic optimization and optimal control, fundamental for the analysis of complex economic models, through the application of the Pontryagin maximum principle and optimality conditions. Finally, it intends to introduce students to the concepts of measure and integration theory, with particular attention to the Lebesgue measure and its properties, to foster a deep understanding of the advanced mathematical tools used in economics, quantitative finance, and social sciences.

These skills are closely linked to the "Statistics" learning area of the Master's Degree in Statistical and Economic Sciences, as they provide the mathematical and analytical foundations necessary for advanced statistical analysis, data modeling and interpretation of complex phenomena, promoting an integrated approach between mathematical theory and statistical applications.

Students will acquire theoretical and practical skills, developing the ability to apply this knowledge to real problems, critically interpret results and propose methodologically sound solutions. The training path promotes independent judgment in the use of mathematical and statistical tools, allowing students to become more confident and independent professionals. The course also contributes to consolidating learning and updating skills in the field of advanced methods of mathematical and statistical analysis, in line with the objective of lifelong learning.

Contents

The contents consist of three parts. The first and the second one are strictly intertwined, whereas the third one, besides connections with the second part,

provides useful notions for such courses as Finanacial Mathematics M.

In the First Part, basic elements of the theory of ordinary differential equation systems are provided.

In the Second Part, an approach to (continuous time) optimal control problems is presented, along with a solution existence result.

In the Third Part, basic elements of measure theory and of integration theory are provided. As a special case, the Lebesgue integral is considered, with special emphasis to convergence theorems (monotone and dominated).

Detailed program

Part I (ODE):

- Differential equations in mathematical economics, Cauchy problems and related solution notion.
- Reduction to first order ODE of higher order ODE.
- Solving explicitly classes of differential equations: separable equations, linear equations, Bernoulli's equations, homogeneous equations, exact equations.
- Application to specific models (market price dynamics, Solow model of economic growth).
- Global and local solution existence and uniqueness for a Cauchy problem.
- Equilibria and their stability (in the Lyapunov sense, local and global asymptotic).
- Elements for a qualitative analysis of autonomous ODE.
- Linear ODE systems: solution methods and stability.

Part II (Optimal control):

- Problem statement.
- The Pontryagin maximum principle (the linear dynamics case and beyond).
- Sufficient optimality conditions (Mangasarian condition and Arrow condition).
- Applications to economical models (optimal selling strategies, selling maximization).
- The simplest problem of the calculus of variations as a special optimal control problem and its application (an optimal consumption/investment model).
- Existence of an optimal control (Filippov's theorem).

Part III (Selected topics in measure theory):

- Algebra and ?-algebra, generated ?-algebra.
- Measures and their properties.
- The Lebesgue measure on **R**n.
- Measurable functions and their properties.
- Integral over a measure space and its properties.
- Integral functions and absolutely continuous functions.
- Convergence theorems (Lebesgue's dominated convergence theorem and B. Levi's monotone convergence theorem).
- Riemann vs Lebesgue integral.

Prerequisites

No official prerequisite. Nevertheless, a refreshement concerning the following topics in Mathematics, typically

learnt in basic courses of calculus for undergraduate students, is strongly advised:

- Basic notions about complex numbers;
- Integration of functions of one real variable;
- Multivariable calculus;
- Matrix calculus with basic elements of linear algebra;
- Eigenvalues and matrix diagonalization methods;
- Quadratic forms;
- Convexity/concavity for sets and functions.

Teaching methods

All lessons are held in person in the following delivery mode:
10 lessons of 2 hours and 6 lessons of 2 hours, all delivered in person.

During the teaching period, some exercises will be proposed to be autonomously solved by students, in preparation of the exam. Some of them will be then discussed in special sessions by the teacher.

Assessment methods

The exam will be in written form and, in case of passing the written test with a sufficient grade (>=18/30), in oral form upon request of the student or the teacher. There are no partial tests in progress.

The format of a written test essentially includes the following types of questions:

- the resolution of 3 exercises/problems;
- a detailed discussion of one of the models presented in the course;
- the detailed exposition of some arguments of the theory and their application in specific cases (open questions).

In carrying out an exam, the ability to analyze and classify a proposed problem, the ability to choose and apply the resolution methodologies proposed in the theory, the depth, precision and completeness of the exposition in the discussion of models and of the theoretical apparatus developed during the course will be evaluated.

Specific material for exam simulations will be provided by the teacher.

Textbooks and Reading Materials

Lecture notes and exercises are provided during the course.

Some further reading:

1. **A. Guerraggio - S. Salsa, *Metodi matematici per l'economia e le scienze sociali*, G. Giappichelli Editore, Torino, 1997.**
2. **K. Sydsæter - P. Hammond - A. Seierstad - A. Strøm, *Further Mathematics for Economic Analysis*, Prentice Hall, Harlow, 2008.**

Semester

The course is scheduled in the second half of the second semester.

Teaching language

Italian.

Sustainable Development Goals

QUALITY EDUCATION | RESPONSIBLE CONSUMPTION AND PRODUCTION