UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Numerical Linear Algebra

2526-1-F4002Q023

Aims

The main goals of the course are:

Knowledge and understanding. The student will learn the fundamental **Singular Value Decomposition** and its application. The state-of-the art algorithms for eigenvalue computation & solution
of linear systems. In particular: iterative solution methods of Krylov subspace type and Randomized algorithm

Applying knowledge and understanding. By means of several examples and exercises, the student will develop the ability of constructing and analysing numerical linear algebra algorithms (for both eigenvalue computation & solution
of linear systems)

Making judgements. The student will be able to face critically problems concerning numerical linear algebra, identifying by himself/herself the most appropriate tools among those introduced in the course. He will be able to interpret and analyse numerical results

Communication skills. The student will become familiar with the introduced language and mathematical formalism, which will make him/her able to communicate with rigor and clarity the acquired knowledge.

Learning skills. The student will be able to apply the acquired knowledge to different applications coming from physics, partial differential equations, data science and Machine Learning which require a good numertical linear algebra background.
The student will be able to interpret and analyse numerical results

Contents

This course builds on elementary linear algebra and in it we derive, describe and analyse a number of widely used constructive methods (algorithms) for various problems involving matrices. Numerical Methods for solving linear systems of equations, computing eigenvalues and singular values and various related problems involving matrices will be the main focus of this course.

We will present different approaches (depending on the size of the matrices,-small,large, huge-), for designing and analzying solution techniques. Namely, direct methods, iterative methods and randomized methods.

We shall consider the subject from mathematical point of view, studying how to construct modern computational algorithms, exploring their properties and validating the algorithms in concrete problems.

Detailed program

0- Introduction. Recap on Linear Algebra concepts and basic matrix decompositions.
1- SVD and its properties. Applications of SVD. Truncated SVD. Randomized SVD
2- Stability. Perturbation Theory. Backward Stability & error analysis
3- Direct methods for eigenvalue problems. Recap of direct methods for Linear Systems & LS

4- Iterative Methods. Krylov subspace methods (for linear systems and eigenvalue problems)

6- Randomised algorithms (for linear systems & least squares). Sketching.

7- Further Applications in Data Mining, Pattern recognition and Machine Learning.

Prerequisites

Solid knowledge of Analysis, Linear Algebra and basic Numerical Analysis.
Solid knowledge of MATLAB
Desiderable: Good knowledge of basic probability

Teaching form

Lectures in class and in the Lab.

We will use MATLAB for all computer examples, exercises and projects.

The students will be given the possibility of adhering to the use of "flipped classroom" (or inverse-blended teaching)for a few of the topics of the course. This option will be completely optional.

Textbook and teaching resource

Different material (slides and notes)will be provided during the course. The course has a big practical component for which we will use MATLAB

We will use several books(several chapters in each of them to cover the different topics)

Bibliography:

-Lars Elden, Matrix Methods in Data Mining and Pattern Recognition, SIAM (2019)
-N. J. Higham, : Accuracy and Stability of Algorithms, (2002)
-Horn and Johnson: Matrix Analysis (2012)
-D. S. Watkins, Fundamentals of Matrix Computation, Wiley, (2002)
-D. S. Watkins, The Matrix Eigenvalue Problem: GR and Krylov Subspace Methods, SIAM (2007)
-J. Demmel: Applied Numerical Linear Algebra (1997)
-Per-Gunnar Martinsson &J. Tropp, Randomized Numerical Linear Algebra: Foundations & Algorithms (2021)
-G. Strang, Linear Algebra and Learning from Data, SIAM, (2019)
-I. Ipsen, Numerical Matrix Analysis, SIAM, 2009

-Golub-Van Loan : Matrix Computations (2012)
-Trefethen-Bau: Numerical Linear Algebra (1997) (also 2022)

Semester

First semester

Assessment method

The evaluation of the course has two parts:

1- the development of a small project (on a topic related to the course and chosen by the student/students)
2- a small (oral or written) exam. Specifics on the oral or written exam will be given at the begining of the course.

The students who adhere to the flipped classroom will have the extra vote from their exposition.

The small project could be chosen from a list of projects that will be made available to the students towards the end of the course. The projects will either require students to delve deeper into one of the topics of the course or try to apply many of the techniques for some particular application that can be written as a matrix problem. Students are encouraged to work on the project **in groups** of at most two or three people. The project should be handed four days before the before the date of the small exam. Part of the small exam will be devoted to the discussion of the project, allowing to validate the knowledge and capabilities of the students related to the course.

Office hours

By appointment (that should be fixed by writing an email to me)

Sustainable Development Goals

QUALITY EDUCATION | INDUSTRY, INNOVATION AND INFRASTRUCTURE