UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Elementary Mathematics

2526-1-F4002Q005

Aims

The aim of this course is to present some elementary results in Number Theory, Topology, Geometry, and Combinatorics. The term "elementary" should be interpreted to mean that no particular prerequisites are required. The presentation of these results is progressive, emphasizing how the introduction of the topics and preliminary problems can be easily understood by high school students. Subsequently, these same problems are developed to reach a deep and modern level of mathematics.

This progression also serves to demonstrate examples of topics that can be presented and understood by a class of high school students, without neglecting a thorough exploration of mathematics for a more complete treatment.

Here are the aims of this course according to the Dublin Descriptors:

1. Knowledge and understanding
   Students will acquire knowledge of fundamental concepts and results in number theory, topology, geometry, and combinatorics. They will understand how seemingly simple problems can lead to deep mathematical insights and how such topics can be introduced progressively, even at the high school level.

2. Applying knowledge and understanding
   Students will learn to apply elementary but rigorous mathematical tools to a variety of problems across different areas of mathematics. They will develop the ability to recognize structures and patterns that connect elementary notions with advanced results and to use these tools in the development of seminar presentations.

3. Making judgements
   Through the analysis of problems and their progressive development, students will learn to critically evaluate mathematical arguments and choose appropriate methods for solving complex problems. They will also assess the depth and clarity of various mathematical presentations and approaches.

4. Communication skills

The seminar-based assessment will strengthen students' ability to clearly present mathematical ideas, both orally and in writing, using appropriate terminology and logical structure. They will learn to explain complex ideas in an accessible way.

5. Learning skills
   By working through problems that evolve from basic to advanced, students will enhance their independent learning skills. They will develop the ability to study autonomously, identify suitable topics for deeper investigation, and engage with mathematical literature at various levels.

Contents

Prime numbers, combinatorics, topology.

Detailed program

1. Prime numbers: density, Bertrand postulate, Basel postulate, Willans' formula, Dirichlet theorem
2. Ramsey theory: applications in combinatorics, geometry and analysis,
3. Applications of topology,
4. Results from geometry, as Minkowski's lemma, to solve questions on integers: sum of squares,
5. Problems arising from probability,
6. Polya enumeration method.

Prerequisites

The prerequisites are the undergraduate courses. Observe, that the word "elementary" should not be understood as simple. The topics are simple and easily understood by anyone and a first analysis of the problems and arguments is also simple.

Teaching form

In-person, lecture-based teaching. In particular, the teaching includes lectures with expository instruction conducted in person. The lectures will be recorded, and the recordings will be made available on the course's e-learning page.

Textbook and teaching resource

Notes of the course given during the lectures.
P.Cameron, Combinatorics, topics, techniques, algorithms, Cambridge university press,
G. Travaglini, Numbers and Figures, American Mathematical Society (2023).
M. Bramanti, G. Travaglini, Studying Mathematics: The Beauty, the Toil and the Method, Springer (2018).

H. Steinhaus, One Hundred Problems in Elementary Mathematics, Dover Publications, 1967.
H. Steinhaus, One Hundred Problems in Elementary Mathematics, Dover Publications, 2011 (reprint).

Semester

First semester

Assessment method

The exam consists of a 45-60 minute seminar on a topic chosen by the student and approved by the instructor. The choice of the topic is free, but it must be compatible with the subjects covered during the course. Additionally, it should present the same gradual increase in difficulty as presented in the lectures.

The clarity of the presentation and the knowledge of the material will be evaluated.

The exam grade is on a scale of thirty, with a minimum passing grade of 18/30. There are no partial exams.

Office hours

by appointment

Sustainable Development Goals

QUALITY EDUCATION