COURSE SYLLABUS

Harmonic Analysis

2526-1-F4002Q001

Aims

The aim of the course is to illustrate some basic material in Fourier analysis, of wide use in analysis and applications. Applications to signal processing will be given. At the end of the course the student will be able to understand the basic issues concerning the theory of signal processing, with emphasis on musical applications. No specific knowledge of musical theory is assumed.

Specifically, the expected learning outcomes include:

- the knowledge and understanding of the fundamental definitions and statements, as well as of the basic strategies of proof in Fourier Analysis and Signal Processing, focusing on the convergence of Fourier series and integrals (pointwise, in mean, uniform), the treatment of the Discrete Fourier Transform and the Fast Fourier Transform, and the diffusion of sound waves;
- the skill to apply such conceptual background to the construction of concrete examples and to the solution of exercises, ranging from routine to challenging (starting with routine exercise that require straightforward application of the definitions and the results given during the lectures, up to exercise that require deep understanding of the matter and the ability of developing original ideas).

Description based on the Dublin Descriptors
? 1. Knowledge and understanding

The student will acquire an in-depth understanding of the theoretical foundations of Fourier analysis, including pointwise, uniform, and mean convergence of Fourier series and integrals, the discrete Fourier transform (DFT), the fast Fourier transform (FFT), and basic aspects of wave propagation and signal modeling. These topics will be addressed with mathematical rigor and situated within the broader framework of harmonic analysis, with applications especially in physics and music.
? 2. Applying knowledge and understanding

The student will be able to apply theoretical results to the analysis and synthesis of signals, the solution of complex problems, and the construction of mathematical models in interdisciplinary contexts (e.g., physics, sound

processing, engineering). The student will also be able to design and implement Fourier transform algorithms, taking into account both conceptual and computational aspects.
? 3. Making judgements

The student will develop critical thinking skills to assess the appropriateness and limitations of harmonic analysis methods. They will be able to choose suitable theoretical tools for specific problems, even in nonstandard contexts, and evaluate the mathematical assumptions underlying the models used.
? 4. Communication skills

The student will be able to clearly, rigorously, and effectively communicate the mathematical content of harmonic analysis, both to specialists and to professionals in related fields (e.g., music, physics, engineering). They will be able to write technical reports and present both theoretical and applied topics, possibly supported by digital tools.
? 5. Learning skills

The course will equip the student with the ability to further explore harmonic analysis and its applications independently. This includes advanced topics such as the generalized Fourier transform, tempered distributions, spectral analysis, and digital signal processing. The student will be able to read and understand specialized literature and research papers, and to build upon this knowledge for further study or research activities.

Contents

Basics on Fourier series and integrals. Applications to signal analysis, and to music.

Detailed program

- Orthonormal systems and the Vitali-Dalzell criterion
- Basic properties of Fourier series in one variable. Mean and pointwise convergence (Dini's test and Jordan's theorem). The Cesaro means and their pointwise convergence. Applications to the vibrating string.
- Fourier analysis in the unit disc of the plane and applications to the stationary waves of the drum.
- The Fourier transform in one variable. The Schwarz space. Plancherel and inversion formulae.
- The Fourier transform in several variables. The wave equation and propagation of sound and the associated Cauchy problems. Spherical means.
- The discrete and the fast Fourier transforms.
- The Paley-Wiener theorem, Poisson's summation formula and the sampling theorem.
- The Gabor transform and spectrograms
- Applications to music and the digitalization of sound.

Prerequisites

In order to be able to successfully attend the course, the student should know basics of Analysis in and Linear Algebra: calculus for functions of several viariables, pointwise and uniform convergence of series of functions, the Lebesgue integral and basics of measure theory, matrix calculus. A knowledge of the main properties of the space L2 and of the elementary theories of Hilbert spaces and holomorphic function will be valuable.

Students lacking prerequisites are invited to contact the professor by e-mail. He will give them bibliographical suggestions useful to fill the gaps and possibly provide further support.

Teaching form

56 hours of in-person, lecture-based teaching (8 ECTS), with blackboard.

The teaching hours will be dedicated either to the illustration of the main results in the theory, or to the solution of problems (previously assigned) of applications of the theory.

The course will be held in Italian.

Textbook and teaching resource

The Lecure notes of the course are availatble on the e-learning page of the course. They contain all the material that will be illustrated during the lectures, together with many exercises. Amongst them the student will find some of the problems contained in the final tests in recent years.

Material for further reading can be found in the following books:

- Stein-Shakarchi, Fourier Analysis, Princeton University Press
- Steiglitz, A Digital Signal Processing Primer, Princeton University Press
- D. Benson, Music: a Mathematical Offering, available (free) at https://homepages.abdn.ac.uk/d.j.benson/pages/html/music.pdf

Semester

II semester.

Assessment method

There will be one written mid-term exam. Its structure will be similar to that, described below, of all other written examinations.
Written examination, including theoretical questions (proofs of part of the results illustrated during the course) and exercises, often similar to those solved during the class hours. In order to get a positive grade, both the parts including theoretical questions and exercises must get a passing grade. The two parts of the written examination will contribute in the same amount to the determination of the final grade.

The grade will take into account the exactness of the answers, the clarity of the exposition and the command of mathematical language used.

Office hours

Upon appointment.

Sustainable Development Goals

QUALITY EDUCATION