UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Representation Theory

2526-1-F4002Q025

Aims

The course is aimed to present the contents and the fundamental methods, as well as some noteworthy applications of the ‘classical’ theory of representations of finite groups.

The expected learning outcomes include:

- the knowledge of the main results in the representation theory of finite groups,
- as well as the ability to apply them on concrete examples.

Contents

Semisimple rings and modules. Modules and representations. Characters of finite groups. Tensor products of representations. Permutation representations and applications. Direct products. Induction and restriction of representations. Clifford Theory.

Detailed program

Semisimple rings and modules.

Generalities on rings and modules. Artinian and noetherian rings and modules. Semisimple rings and modules. Simple modules. Decomposition of a semisimple modules in isotypic components. Structure of semisimple rings. Wedderburn’s theorem. Double centralizer property (DCP). Structure of simple artinian rings.

Modules and represesentations.**

The group algebra KG. KG-modules and G-representations. Completely reducible representations. Maschke's theorem. Representations over splitting fields: structure of KG. Frobenius-Schur theorem. Examples of complex representations of finite groups.

Characters of finite groups.**

Definition and properties of characters of a group G. The space CF(G) of class functions. CharK = 0 and K splitting for G: characters and modules; the character table. Regular representation, orthogonal idempotents, first orthogonality relations. Irr(G) is an orthonormal basis of CF(G); second orthogonality relations. Algebraic integers and characters; structure constants of the centre of KG. The degree of an irreducible character divides the order of G. Applications:The p^a.q^b Theorem of Burnside. Structural properties of a group detectable from the character table [Remarks on representations of compact groups]

Tensor products of representations.

Tensor products of modules. Tensor products of representations, products of characters. The ring of virtual characters. The Burnside-Brauer theorem. Counting involutions, the Brauer-Fowler theorem and its implications.

Permutation representations and applications.**

Permutation groups. Actions on conjugacy classes and characters. Brauer's permutational Lemma. Real characters.

Direct products.**

Irreducible characters of a direct product. Application: Burnside's theorem on character degress.

Induction and restriction of representations, Clifford theory.**

Representations induced from subgroups. Induced characters. Frobenius reciprocity law and applications. Restriction to a normal subgroup: Clifford's theory. Inertia group, Clifford correspondence. Ito's theorem.

Prerequisites

It is recommended an a priori knowledge of the standard contents of first and second year Algebra courses, plus some extra knowledge of field theory.

Teaching form

56 hours of lessons in presence given in English (8 ECTS) using a white or black board.

Textbook and teaching resource

C. W. Curtis and I. Reiner, Representation Theory of Finite Groups and Associative Algebras, Wiley Interscience 1962.

C. W. Curtis and I. Reiner, Methods of Representation Theory I, Wiley 1981.

L. Dornhoff, Group Representation Theory, Marcel Dekker 1971.

B. Huppert, Character Theory of Finite Groups, de Gruyter 2011.

I.M. Isaacs, Character theory of finite groups, Academic Press 1976

Semester

1st semester

Assessment method

The exam is only oral. It consists of a number of questions and an evaluation (marks: 18/30 to 30/30). The questions are aimed to verify that the student has understood the theoretical development of the course and has a good knowledge of the theorems (and their proofs), as given in the lectures.

Office hours

On appointment.

Sustainable Development Goals

QUALITY EDUCATION