UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Topological Methods in Field Theories

2526-1-F4002Q035

Aims

According to the educational objectives of the Course, the taught material aims to provide students with the *basic notions* regarding the definitions and the fundamental results for a geometric and topological approach to the study of classical field theory, with particular emphasis on classical vortex dynamics, ideal magnetohydrodynamics and quantum hydrodynamics. The course aims to provide also the necessary *competences* to understand and use standard techniques and the demonstration methods involved in the theory, as well as the *capabilities* to use them to solve exercises and tackle problems.

1. Knowledge and understanding

The student will acquire a clear and systematic understanding of the fundamental definitions and statements, as well as of the basic strategies of proof in classical geometric and topological field theory; the knowledge and understanding of some key examples where theory is fully applied.

2. Applying knowledge and understanding

The student will be able to recognize the role that concepts and techniques from a geometric and topological approach play in various areas of mathematics (such as vortex theory, ideal magnetohydrodynamics, quantum fluids), and in the mathematical modelling of physical situations (vortex dynamics, relations between energy and complexity, topological defect production, knotting and linking); skills to apply the basic concepts to the elaboration of practical examples and to the solution of posed questions; the ability to communicate and explain in a clear and precise manner both the theoretical aspects of the course as well as their applications to specific situations, possibly in analogous but different contexts.

3. Making judgements

The student will develop the ability to critically interpret definitions, statements, and proofs, identifying the most appropriate conceptual tools for analysing and solving proposed problems.

4. Communication skills

The student will be able to present the fundamental concepts of the course clearly and rigorously, using correct mathematical language.

5. Learning skills

The student will develop the learning skills needed to pursue further studies in mathematical analysis and related fields, including the ability to consult scientific texts and appropriate educational resources independently.

Contents

This course aims to provide the fundamentals to apply topological techniques to the study of classical theories of knotted fields. The course contents are the following:

Fundamentals of Green's potential theory, fluid flows and diffeomorphisms, conservation theorems of Euler's equations, Helmholtz's conservation laws, ideal magnetohydrodynamics, kinetic and magnetic helicity, potential theory in multiply-connected domains, elements of knot theory, linking number and self-linking, topological interpretation of helicity, geometric decomposition, relaxation of magnetic knots, topological defects in condensates, Hopfions and knotted fields, change of topology by reconnection processes.

Detailed program

The course is divided into two parts, the first being of introductory and general character, the second being more focused on specific topics of current research. In particular:

Fundamentals of potential theory in terms of Green's identities, fluid flows and diffeoporphisms, kinematic transport theorem, conservation theorems, decomposition of fluid motion, Euler's equations, vorticity transport equation, Helmholtz's conservation laws, Biot-Savart law, Maxwell's equations, ideal magnetohydrodynamics, kinetic and magnetic helicity, perfect and non-perfect analogous Euler's flows.

Kelvin's correction for multiply connected domains, elements of knot theory, fluid dynamic interpretation of Reidemeister moves, inflexional configuration and twist energy, linking number and self-linking, derivation of linking number from magnetic helicity, writhe and twist helicity, relaxation of magnetic knots, groundstate energy spectra, hydrodynamics interpretation of Gross-Pitaevskii equation, topological defects in condensates, Hopf map, Hopfions and knotted fields, change of topology and physical surfaces, reconnection processes, knot polynomial invariants, measures of topological complexity.

Prerequisites

Elements of differential geometry of curves and surfaces in three-dimensional space, elements of mechanics of continuum systems, differential operators in mathematical physics and balance laws in physics.

Teaching form

Standard lectures on blackborad taught in English and supported by lecture notes (in English) made available to the students.
A hybrid teaching approach is used, that combines lecture-based teaching (DE) and interactive teaching (DI). DE involves detailed presentation and explanation of theoretical content. DI includes active student participation through exercises and problems, short presentations, group discussions, and group or individual work. It is not possible to precisely determine in advance the number of hours dedicated to DE and DI, as these methods are dynamically intertwined to adapt to the course's needs and promote a participatory and integrated learning environment, combining theory and practice.

Textbook and teaching resource

Lecture notes (in English) made available by the lecturer during the course.

Semester

II semester.

Assessment method

Oral exam (in Italian or English) based on 4 questions taken from a list of questions made available to the student at the end of the course. Specific solutions must reproduce the material presented during the course, including detailed proofs of theorems and statements, complete with explicit computations. The final mark is expressed in 30 units.

The written examination paper must show operational *capability* to tackle and solve the proposed questions by using the acquired *knowledge* and the necessary *competence* to reproduce the topics presented during the course.

Office hours

Upon appointment, to be arranged with the lecturer by email contact: renzo.ricca@unimib.it.

Sustainable Development Goals

QUALITY EDUCATION