UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Metodi della Fisica Matematica

2526-1-F4002Q019

Aims

The course presents the physical and mathematical foundations of classical field theory, dealing especially with Fluid Dynamics. The ideas, and the fundamental equations will be presented. Some applications and examples will be dealt with in details. A relevant part of the course deals with wave motion: linear and nonlinear waves, dispersionless and dispersive waves, and soliton equations.

The main expected learning outcomes are:

1) **Knowledge and understanding:**
The knowledge and understanding of the definitions of "continuum mechanics" and, especially, fluid mechanics; the knowledge of the physical motivations thereof, of the main theoretical results and of the basic strategies for their proofs.

2) **Applying knowledge and understanding:**
The mastering of the different approximations needed in the modelling processes (such as constitutive equations, linearization processes, asymptotic expansions) discussed during the course.

3) **Making judgements:**
The ability to apply the acquired conceptual background in the analysis of the various applications, and the ability of choosing the right token from such a "toolbox".

4) **Communications skills:**
The acquisition/improvement of the skill in presenting and clearly discussing both the theoretical contents of the matter and their implementation in specific situations, possibly related with a broader scientific area.

5) **Learning skills:**
The skill to build on the acquired knowledge by further refinements to be used in the analysis of subjects not fully developed during the lectures.

Contents

- The configuration space for continous bodies.
- Stress and Deformation tensors. Velocity gradient.
- Transport theorems and their geometrical formulation in terms of differential forms in the Euclidean three-space.
- The mass conservation equations, the Cauchy equations, the energy equation and the entropy inequality.
- Isotropic stresses and the Euler equations.
- Static and stationary solutions.
- Sound waves and incompressibility.
- Bernoulli's theorem and applications.
- Hemholtz equations.
- Theory of aerofoils.
- Viscous stresses and the Navier-Stokes equations. First applications.
- Scalings and the Reynolds number. Boundary layers and Prandtl equations.
- Gravity waves: air-water and stratified fluids. Surface tension.
- Small-amplitude gravity water waves
- Waves in shallow water: the Korteweg-de Vries (KdV), Burgers, Airy and noon-linear Schrödinger equations. Solitons.
- Hamiltonian formulation of the KdV equation.
- The KdV equation as a completely integrable Hamiltonian system.

Detailed program

The starting point of the course is the analysis of the deformation and of the motion of a continuous body, through the introduction of the notions of deformation gradient and velocity gradient. This part of the course introduces and makes use of methods of differential geometry in the Euclidean three space. Transport theorems of scalar and vector quantities are then discussed and proved as relevanmt part of the kinematics of continuum bodies.

Then dynamics is considered via the study of the external actions on a continuum deformable body. The core is Cauchy's stress theory. The mass conservation law and the balance equations for linear and angular momentum and energy are discussed. The notions of internal energy and entropy are discussed.

The mechanical (and thermal) properties of fluids are characterized via the constitutive and state equations. Models of elastic fluids (both in the compressible and incompressible regimes) and possibly viscous fluids are considered.

Then the course focusses on the motion equations starting from the Euler model.

Starting from static solutions, we move to an ample section devoted to the study of the Euler equations for the so-called ideal fluids and its consequences and applications. The most relevant are the Bernoulli equation, the Helmholtz laws on the vorticity evolution and Kelvin's circulation theorem.

Then the aerofoil theory of Kutta-Joukowski is presented.

Then the most relevant properties of viscous fluids, described by the Navier Stokes equation, are studied.

Then the following points are introduced:

- the transport of linear momentum through shear stresses and the non-conservation of mechanical energy in the Navier-Stokes theory

- Scale transformations, self-similarity and the Reynolds number.
- The boundary layer and the Prandtl equations.

Then we move on to a more "applicative" section of the course, mostly dedicated to the theory of water waves, to be developped along the following points (time permitting):

- Gravity waves in an incompressible fluids (surface waves).
- Gravity waves in stratified fluids (internal waves).
- Gravity waves in the presence of surface tension.
- Gas dynamics and quasi-linear equations: theory of characteristics and shock waves.
- Small-amplitude gravity waves on the surface of deep water and the Nonlinear Schrödinger equation.
- The Korteweg-de Vries equation for waves in shallow water: non-linearity and dispersion. Solitons and cnoidal waves.
- The Hamiltonian formulation of KdV and the constants of the motion.

Prerequisites

No course of the Master Degree in Mathematics is strictly required for attending the present course. The basic notions of the courses Mathematical Analysis I and II, Linear algebra and Geometry, Physics I and II and Dynamical Systems and Classical Mechanics of the Batchelor Degree are needed.

Teaching form

Lectures (8CFU), expository teaching. Students will attend lectures where the instructor will present theoretical material, demonstrate problem-solving techniques as well as apply the latter to specific examples.
We foresee to deliver lectures in the Italian language. However, we may switch to English, should non-italian students be attending.

Textbook and teaching resource

Reference texts:

1. A.J. Chorin, J.E. Marsden: A mathematical introduction to fluid mechanics, Springer 2000.
2. S. Salsa: Partial Differential Equations in Action: from Modeling to theory. Springer, 2008.
3. G. Falkovich, Fluid Mechanics (a short course for physicists). Cambridge University Press, 2011.

The notes of the lectures are published in the e-learning page.

Semester

First semester.

Assessment method

The first part of the examination consists in the presentation of a written homework on a subject chosen within a list provided by the end of the lectures by the instructor. The list will comprise (also) items complementary to those discussed in the lectures. The student should inform the instructor about her/his choice of the subject of the homework at least 10 days befor the discussion date. Also, she/he must send a copy of the homework to the instructor at least 2 days before that date for a preliminary evaluation.

The main aim of this first part mainly regards points 3 and 4 of the above-mentioned "expected learning outcomes". The evaluation will regard, also taking into account the complexity of the chosen homework subject, the clarity of the exposition, the ability to sinthetyze the subject as well as the degree of mastering of the subject acquired by the student.

In the second part, the student will be asked to discuss a few of the main points of the program (at the instructor's choice). This part mainly addresses points 1 and 2 of the "expected learning outcomes".

For what the exam's outcome is concerned, the relative weight of the two parts is equal.

Office hours

Meetings whose schedule is to be agreed either via e-mail (preferred) or this e-learning page.

Sustainable Development Goals

QUALITY EDUCATION