UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Analisi Reale ed Equazioni Differenziali

2526-1-F4002Q003

Aims

Knowledge and understanding
The course provides a solid theoretical background in linear partial differential equations (PDEs), with introductory elements on nonlinear equations, and develops understanding of the main analytical results, proof techniques, and methods in advanced mathematical analysis.

Applying knowledge and understanding
Students will be able to apply the theoretical concepts to solve exercises, analyze models, and rigorously formulate differential problems, also in interdisciplinary contexts.

Making judgements
The course fosters critical thinking and autonomy in selecting appropriate analytical tools for modeling and theoretical problems, as well as in validating the obtained results.

Communication skills
Students will develop the ability to clearly and rigorously present topics and proofs related to real analysis and PDEs, using formal mathematical language both in written and oral form.

Learning skills
The course promotes the development of an autonomous and advanced study method, essential for reading specialized literature and for continuing with research or high-level scientific applications.

Contents

Spectral theory for compact and selfadjoint operators. Elliptic equations: regularity, maximum principles, eigenvalues and eigenfunctions of the Laplacian. Transport equation and characteristics. Bochner's integral.

Parabolic partial differential equations: weak solutions, Galerkin method, energy estimates and maximum principle. Introduction to the theory of evolution semigroups in Banach spaces.

Detailed program

Second order elliptic equations: regularity of weak solutions, weak and strong maximum principles.
Spectral theory: adjoint, selfadjoint, compact operators, spectrum. Spectrum of compact operators. Fredholm alternative. Spectral decomposition of selfadjoint compact operators. Eigenvalues and eigenfunctions of the Laplacian.
Transport equation: The method of characteristics.
Bochner integral: Definition, main properties and Sobolev spaces defined with the Bochner integral.
Parabolic equations: Weak solutions for second order parabolic equations. Galerkin's method. Energy estimates, existence and uniqueness of weak solutions. Maximum principle.
Introduction to the theory of evolution semigroups in Banach spaces: Uniformly continuous and strongly continuous semigroups, basic properties and their generators. Resolvent set and resolvent operator. Contraction semigroups and properties of their generators. The Hille–Yosida theorem.

Prerequisites

Banach and Hilbert spaces, **L**? spaces, their duals and properties, Sobolev spaces and immersion theorems.

Teaching form

56 hours of classroom lessons, delivered didactics (8 ECTS credits)

Course delivered in Italian with the possibility of being delivered in English if foreign students request it.

Textbook and teaching resource

- A. Bressan. Hyperbolic systems of conservation laws: the one-dimensional Cauchy problem. Vol. 20. Oxford University Press on Demand, 2000.
- A. Bressan. Lecture Notes on Functional Analysis. With applications to linear partial differential equations. American Mathematical Society, 2013.
- H. Brezis. Functional analysis, Sobolev spaces and partial differential equations. Springer Science and Business Media, 2010.
- L. C. Evans, Partial Differential Equations, AMS Graduate Studies in Mathematics, Vol.19. Second Edition, Providence 2010.
- D. Gilbarg, N. S. Trudinger, Elliptic partial differential equations of second order, Reprint of the 1998 edition. Classics in Mathematics. Springer-Verlag, Berlin, 2001.

Course webpage: https://elearning.unimib.it/course/view.php?id=62151

Semester

Second semester.

Assessment method

The exam consists of a written examination and an oral discussion.

The written examination consists in a short essay. The student is asked to develop two topics out of four proposed in two hours. The first chosen topic must be related to the first part of the course and the second topic to the second part. The exposition must be precise, detailed, exhaustive and consistent with the chosen topics and must contains some of the most significant proofs. The ability to present a selection of significant proofs and, above all, the critical and operational knowledge of the definitions and results presented during the course, also by the illustration of examples and counter-examples are evaluated.

The oral discussion is held a few days after the written examination and consists in a short discussion and correction of the short essay. It verifies the mastery of the chosen topics. No other topics or proofs are asked outside of the two chosen topics.

Final grade out of thirty.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION