UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Stochastic Methods and Models

2526-1-F4002Q028

Aims

The "Stochastic Methods and Models" course aims to provide students with an in-depth selection of advanced methods, concepts, and models from probability theory and stochastic processes, from both a theoretical and practical point of view.

Upon completion of the course, students will have acquired the following learning outcomes, expressed according to the Dublin Descriptors:

1. Knowledge and understanding:
   Students will possess deep and specialized knowledge of advanced results in probability theory (large deviations), stochastic processes (continuous-time Markov chains), and stochastic modeling (random graphs). They will be able to formally understand and interpret complex concepts related to rare events, Poisson processes, random walks, and properties of random graphs.

2. Applying knowledge and understanding:
   Students will be able to apply the acquired advanced theoretical notions to solve complex exercises and critically analyze stochastic problems and models in both academic and applied contexts. They will have an operational understanding of probabilistic language and master advanced proof techniques, such as coupling.

3. Making judgements:
   Students will be capable of formulating independent and critical judgments on the relevance and applicability of specific stochastic methods and models for analyzing complex phenomena. They will know how to evaluate the validity of assumptions and the implications of results obtained from advanced stochastic models.

4. Communication skills:
   Students will be able to communicate clearly and rigorously, both orally and in writing, complex concepts, analysis results, and proofs related to stochastic methods and models, to both specialists and non-

specialists. The course, being conducted in English, will further enhance these skills in an international context.

5. Learning skills:
   Students will have developed the learning skills necessary to autonomously undertake further studies in fields related to advanced probability, stochastic processes, and mathematical modeling, as well as to stay updated on developments in scientific research within the field. They will be able to delve into specific topics using advanced texts and resources.

Contents

The course starts with an introduction to **large deviations**, a theory that provides tools to investigate the probability of rare events at exponenial scale. In the second part of the course some advanced results on **discrete time Markov chains** are given, as well as an introduction to the **continuous** counterpart. In particular, the **Poisson process** will receive special attention, being a natural example of continuous-time stochastic process having discrete states. In the third part of the course we shall study topics related to **random walks**, a fundamental and rich object in probability. In the last part of the course we will discuss the theory of **random graphs**, a research topic that is receiving great attention.

Detailed program

1. Large deviations

- Cramer's Theorem
- Relative entropy and Sanov's Theorem
- Large deviations principle
- The contraction principle and Varadhan's lemma

2. Discrete & Continuous-time Markov chains

- Reminders (irreducibility, classification of states)
- Markov property
- Invariant measures and convergence to equilibrium
- Semigroups and generators on countable spaces
- Poisson process

3. Random walks

- Simple random walk: path properties for the one-dimentional case, Polya's Theorem
- Random walks on graphs: Harmonic functions, Dirichlet problem, Random walks in random environments
- Recurrence and transience of countable Markov chains: Lyapunov functions and Foster-Lamperti's criteria

4. Random graphs

- Erd?s–Rényi model
- Thresholds in the Erd?s–Rényi model: Connectivity and the emergence of a giant component

Prerequisites

The knowledge, competences and skills taught in classical probability and stochastic processes courses (random variables, martingales, conditional law) as well as those taught in mathematical analysis courses.

Teaching form

The course consists of 56 hours of in-person, lecture-based teaching, equivalent to 8 ECTS. It is divided into two main components:

- **Theoretical:** with focus on presenting definitions, results, and relevant examples.

- **Practical:** with focus on the skills necessary to apply theoretical knowledge to both model analysis and exercise solutions.

The course will be conducted in English.

Textbook and teaching resource

Course's lecture notes by the lecturer.

Reference textbooks:

- F. den Hollander. *Large Deviations*, Fields Institute Monographs, vol. 14. AMS (2008).
- E. Pardoux. *Markov Processes and Applications: Algorithms, Networks, Genome and Finance*, Wiley (2008).
- Q. Berger, F. Caravenna, P. Dai Pra, *Probabilità: un primo corso attraverso esempi, modelli e applicazioni* (II edizione), Springer (2021).
- T. M. Liggett. *Continous time Markox Processes (An Introduction)*, American Mathematical Society (2010).
- G. Last, M. Penrose. *Lectures on the Poisson Process*, Cambridge University Press (2017).
- S. Asmussen, *Applied Probability and Queues*, Springer (2003).
- R. Durrett. *Probability: theory and examples*. 5th edition (2019). The book can be downloaded for free from his personal webpage https://services.math.duke.edu/~rtd/.
- R. Lyons and Y. Peres, *Probability on Trees and Networks*, Cambridge University Press (2016). The book can be downloaded for free from Lyons homepage https://rdlyons.pages.iu.edu/prbtree/book.pdf.

Semester

Spring term

Assessment method

The exam consists of two parts*:* **individual assignment of exercises** contribuiting one sixth to the final grade, and an **oral exam** contribuiting five sixths to the final grade, which will be converted as a 30 point score.

The **individual assignment of exercises** consists in the resolution of some exercises proposed during the course, which have to be solved autonomously by the students and due (at least) 5 days before the oral exam. This examination tests the continuity of learning as well as practical skills.

The **oral exam** consists in an interview lasting about 30-60 minutes and tests the knowledge of definitions, statements and examples presented during the course, as well as presentation skills related to a selection of topics and detailed proofs.

There will be 6 exam sessions (two in June/July, one in September and three in January/February)..

Office hours

By appointment

Sustainable Development Goals

QUALITY EDUCATION