UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Dynamical Systems, Information Theory and Complexity

2526-1-F4002Q026

Aims

The course aims to provide knowledge and skills in the fields of dynamical systems theory, information theory, and algorithmic complexity, as well as the ability to apply these concepts in both theoretical and applied contexts. The learning objectives are structured according to the five Dublin Descriptors:

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1. Knowledge and Understanding

The course provides students with:
• knowledge of the fundamental concepts of modern dynamical systems theory, information theory, and algorithmic complexity;
• skills needed to understand the related proof techniques and to independently explore and deepen some of the above-mentioned topics.

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2. Applying Knowledge and Understanding

Students will acquire the knowledge and abilities necessary to:
• apply the acquired concepts to the study of elementary dynamical systems;
• solve exercises of varying levels of difficulty;
• perform computational analysis of symbolic sequences of different types, with particular attention to applications in the biological and literary domains.

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3. Making Judgements

The course fosters:
• the ability to critically evaluate the presented mathematical techniques and results;
• autonomy in studying and exploring advanced topics;
• reflection on the limitations and potential of the methodologies used.

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4. Communication Skills

By the end of the course, students will be able to:
• clearly and rigorously present theoretical concepts;
• describe problem-solving strategies and mathematical methods;
• use appropriate scientific language in both specialist and general contexts.

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5. Learning Skills

The course develops:
• the skills necessary to pursue further studies in mathematics or in interdisciplinary fields;
• the ability to independently update and expand one’s knowledge;
• the aptitude for problem solving, even in non-standard contexts.

Contents

The course aims to provide the student with an in-depth knowledge of the theoretical framework underlying the analysis of symbolic sequences of different origin. The main contents include: statistical approach to dynamical systems, information sources, algorithmic information content.

Detailed program

The course is divided into three parts:

1. Examples of discrete-time dynamical systems. Elements of topological dynamics. Symbolic dynamics. Ergodic theory. Kolmogorov-Sinai entropy.
2. Shannon entropy. Relative entropy, mutual information. Asymptotic equipartition. Entropy rate for stationary stochastic processes. Codes: Kraft inequality, optimal codes, efficiency of a code. Universal compressors. LZ78 algorithm.
3. Turing machines. Universal machines. Kolmogorov algorithmic complexity. Universal probability. Halting problem. Chaitin's number. Brudno's theorem.

Prerequisites

No course of the Master Degree in Mathematics is strictly required for attending the present course. The only prerequisites are the mathematical knowledge, competences and skills acquired during the three-year grade, especially in the courses of Dynamical Systems and Classical Mechanics, Measure Theory, Probability.

Teaching form

Lectures via expository teaching with blackboard.
Lectures are scheduled in Italian but they could be held in English in the presence of foreign students.

Textbook and teaching resource

There is not a single textbook covering all topics.

Many of the topics are covered by:

- M.Brin & G. Stuck, "Introduction to Dynamical Systems", Cambridge University Press. 2002 (1 copie available in the library; (*e-book online)*
- *P.Walters, “An Introduction to Ergodic Theory”, GTM 89, Springer-Verlag (2 copies available in the library; (e-book not available)*
- T. M. Cover & J. A. Thomas, “*Elements of Information Theory*”, 2nd ed., Wiley-Interscience *(2 copies available in the library; (e-book online)*
- M.Li, P.Vitányi, “An Introduction to Kolmogorov Complexity and Its Applications”, second edition, GTCS, Springer-Verlag, 1997; (*e-book online)*

Lecture notes will be distributed covering all the arguments.

Semester

II Semester.

Assessment method

There are not partial exams. The final evaluation will be an exam (of about 45 minutes) in which the student will be assessed both on mathematical aspects of the theory (definitions, statements, proofs), on the application of the theory (examples discussed during lectures), as well as on the ability to handle the topic independently. Optionally, the student can integrate the exam with the presentation of a project (the choice of the project should be discussed in advance with the instructor). In this case, the relative weight of the project and of the oral examination is equal.

Office hours

Upon appointment.

Sustainable Development Goals

QUALITY EDUCATION