UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Istituzioni di Matematiche

2627-1-G8501R006

Course title

Foundations of mathematics

VERY IMPORTANT

The students of the course Istituzioni di matematiche are subdivided into groups AL (professor Daniela Bertacchi) and MZ (professor Pablo Spiga), according to the initial of the surname. The professor of your group is the one you must refer to for any question.

The professor of your group is the person you must always refer to, addressing any e-mail or communications to Daniela Bertacchi if you are in the AL group, to PabloSpiga if you are in the MZ group.

The enrollment password is
mateal for students AL
matemz for students MZ.

Topics and course structure

This course introduces the basic results in arithmetics and in some other number fields. Moreover, we introduce some elements on sets and on functions and in probability theory.

- Elements of set theory: operations among sets.
- Functions, injective, surjective and bijective functions. Infinite sets.
- Binary relations. Equivalence relations and order relations: equivalence classes and partitions.
- The set of natural numbers. Introduction to the natural numbers via the Peano axioms. Sum, product and order in the natural numbers. Induction principle. Representation of natural numbers in base 10 and in other

bases.

- The integers. Construction of the integers from the natural numbers. Divisibility: quotient and remainder. Congruences mod n, and some modular arithmetic. Prime numbers, fundamental theorem of arithmetic. Eratostene's sieve. Greatest common divisor and minimal common multiple via Euclide's algorithm.
- Rational numbers. Construction of the rational numbers from the integers. Elementary properties of rationals: algebraic and topological properties.
- Basic introduction to probability theory. Independent events and conditional probability. Elementary computation of probabilities.A brief introduction to the construction of the real field from the rationals.

Objectives

Upon completion of the course, the student will be able to:

(Knowledge and understanding)
Understand the fundamental concepts of basic arithmetic (number sets, operations, properties) and elementary logic, also through a formal and non-school-based approach.

(Applying knowledge and understanding)
Consciously apply hypothetical-deductive reasoning and certain tools of logic to analyze mathematical problems, formulate hypotheses, and verify their validity.

(Making judgments)
Critically evaluate the appropriateness of the mathematical tools used in problem-solving and reasoning, with particular attention to the context of teaching in primary school.

(Communication skills)
Clearly and rigorously present mathematical concepts, procedures, and results, using correct mathematical language.

(Learning skills)
Develop a reflective attitude towards mathematics.

Methodologies

Lectures 49 hours, Delivered Didactics (24 2-hours lectures and one 1-hour lecture, in presence).

Exercise classes in interactive teaching online, for a total o 12 hours. These classes are asynchronous and consist in:

- viewing videos of the solution of typical exercises;
- 7 online worksheet, both in PDF form, which are commented in asynchronous mode, and on the interactive online platform wims.

Students who deliver the solutions in the scheduled time will be counted as present during the class.

Some meetings in small groups to tutor students who want some help on the topics covered in the course, take place during the year.

The course is in Italian.

Online and offline teaching materials

Books (see bibliography).

Online: a list of exercises to be solved on paper, and interactive exercises on the platform wims. Asynchronous lectures are present on the elearning platform.

Programme and references

PROGRAMME

Elements of set theory: operations among sets.

Functions, injective, surjective and bijective functions. Infinite sets.

Binary relations. Equivalence relations and order relations: equivalence classes and partitions.

The set of natural numbers. Introduction to the natural numbers via the Peano axioms. Sum, product and order in the natural numbers. Induction principle. Representation of natural numbers in base 10 and in other bases.

The integers. Construction of the integers from the natural numbers. Divisibility: quotient and remainder. Congruences mod n, and some modular arithmetic. Prime numbers, fundamental theorem of arithmetic. Eratostene's sieve. Greatest common divisor and minimal common multiple via Euclide's algorithm.

Textbook M. Cazzola, Matematica per scienze della formazione primaria **Further material*

- G. Caiati - A. Castellano, In equilibrio su una linea di numeri, Mimesis, 2007
- A. Cerasoli, Io conto, Feltrinelli, 2010
- A. Cerasoli, Sono il numero 1, Feltrinelli, 2008
- P. Cereda et al, L'aritmetica del Pirata Newton, Mimesis, 2010
- P. Cereda – G.Dimitolo, La ciurma del Pirata Newton, Mimesis, 2008
- H. M. Enzensberger, Il mago dei numeri, Einaudi

Erasmus
Erasmus students may choose to write the part of the exam which is on paper (open question/exercise), and the oral examination, in English.The programme is the same as for non-Erasmus students.

Assessment methods

- Type of test

The exam consists of two computerized tests (the first denoted as Arithmetic zero and the second denoted Exam test) plus a possible oral test.

1. **Aritmetica Zero Test:** it is a test on arithmetic skills that should be acquired from elementary school. During the test, the use of calculators is not allowed. Registration takes place via the platform

https://wims.matapp.unimib.it/ and the test takes place on the same platform (type of test: OPEN QUESTIONS WITH NUMERICAL ANSWERS). This test must be successfully passed (grade greater than or equal to 21/30) on a date prior to that of the written exam (or on the same date as the written exam if a zero arithmetic session is also activated on that day). The test must be passed, the mark is not evaluated in the final mark of the exam. Students are invited to pass it as soon as possible during the academic year (check for the available dates on the wims portal, under "Questionari" where you can also register). Failure to pass Arithmetic zero before the date of the written test involves not admitting to the written exam. The Arithmetic zero test is valid for one academic year. So if passed in the academic year 202x/2y it is valid until 30/9/202y. (The term of validity rule does not apply to those who took the test before 1/10/2015). More information on the test of Arithmetic zero can be found on the site https://wims.matapp.unimib.it/ inside the page of the course.
The test is 45 minutes long and consists of 8-10 questions.

2. **Exam test:** registration is compulsory on s3w.si.unimib.it. Registration on s3 is also valid for the oral test/registration of the outcome. The exam consists both of exercises similar to those seen in exercise classess and more theoretical questions. The test is computerized on the platform https://wims.matapp.unimib.it/ (where students are invited to practice), plus a part of questions / exercises to be carried out on paper. Type of questions: MULTIPLE CHOICE QUESTIONS and OPEN QUESTIONS WITH NUMERICAL ANSWERS for the part on the wims platform; EXERCISES and/or OPEN QUESTIONS for the part on paper. The duration of the test is approximately 2 hours and cosists of 15-18 questions, and during it *the use of textbooks or notes and handouts is NOT allowed*. The use of calculators is not allowed.

3. **Oral test/verbalization:** The oral exam focuses on the course topics (both those covered during the lectures and on the textbooks, see the programme) and it is about 20 minutes long. The oral exam grade is averaged with the written exam grade. If a student is unable to explain the solutions to the questions answered in the written exam, or refuses to take the oral exam at the time and in the manner required by the instructor, the oral exam grade will be void.

Instructors may decide to exempt certain students from the oral exam or limit it to a brief interview to confirm the grade; however, the oral exam remains mandatory in the following cases:

a) The oral exam is compulsory for those who obtain a mark between 16 and 20 in the written test (including extremes).

b) Those who obtain a mark in the written test greater than or equal to 26: these students can choose whether to accept a mark of 26 or take an oral test in order to obtain a higher mark (it being understood that the oral test, if unsatisfactory, it can lead to a lowering of the final vote). In practice, if a student obtains 27 in the written test, he can decide to record the mark without taking the oral test: in this case the mark 26 will be recorded. Otherwise, the student can decide to take an oral test: depending on the progress of the test Oral, grade 27 can be lowered, confirmed, or raised.

The oral test must be taken in the same session as the written test. In practice, for example, if a student intends to take the written test on the second session of February, then he is also required to take the oral test in the second session of February and no later. If the oral test does not take place in that date, the written test is considered canceled.

4. **Partial tests:** the student of any year of the course can choose to take, in place of the written test, two partial tests (denoted as the first and second "compitino") normally scheduled the first in November and the second coinciding with the first exam session (in January). Partial tests have the same modality as the exam. *To be admitted to the partial tests, attendance of the 7 training sessions is mandatory, in the current academic year*.

5. **Passing the partial tests:** those who obtain in the first partial test a grade greater than or equal to 14 are admitted to the second partial test. The second partial test is passed if a grade greater than or equal to 16

is obtained and the arithmetic mean between the marks of the first and second tests is greater than or equal to 16. There is no oral examination/verbalization after the first partial test, but there is after the second partial test. For the rules regarding the oral examination after the partial tests, see Section 3. **Oral test/verbalization**.

6. **Choice between second partial test and first exam:** the date of the first session in January coincides with that of the second partial test. Therefore, those who pass the first assignment can decide whether to take the second partial or the total test on that date. (Check with the notice forum in December when and how you must tell the decision).

7. **Exam sessions.** The exam sessions are 3: one in January-February with three dates, one in June-July with three dates, and one in September with one date.

8. **Exercise sessions:** rounds of exercise sessions are set up, on a weekly basis, for a total of 7 meetings each. All students interested in attendance are invited to register on the scheduled dates, through the elearning page of the course. Attendance is mandatory for admission to partial tests.

9. **Evaluation criteria:** both in the partial exams and in the regular exams, we evaluate the correctness of the answers, the completeness and the ability to clearly and precisely argue the course topics. The score of the written exam is given by 8/30 for the part on paper (open questions) and by 22/30 for the wims part. The score of the oral exam is averaged with the score of the written part. The structure of the exam is consistent with the objectives of the teaching since it requires the ability to independently solve exercises and to clearly present one's knowledge of the topics.

Evalutazion table
An evaluation table (in Italian) is found here.

Office hours

Office hours by appointment (send an email to your professor):

students AL: Daniela Bertacchi daniela.bertacchi@unimib.it

students MZ: Pablo Spiga pablo.spiga@unimib.it

Programme validity

One year.

Course tutors and assistants

bandi in espletamento

Sustainable Development Goals