UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Relatività

2627-3-E3001Q073

Aims

Contents:

- Detailed study of Einstein's special relativity and of some of its main consequences.
- Covariant formulation of the laws of relativistic dynamics and of the electromagnetism (Maxwell equations).
- Lagrangian formalism and introduction to classical field theory, fundamental prerequisite for quantum-field-theory studies.

The objectives can be schematically divided into five categories:

- Knowledge and understanding: Students will learn the fundamental concepts of special relativity formulated in a covariant (manifestly invariant) way, and will be able to connect this formulation with what they have already learned in previous courses. They will also acquire basic knowledge of classical field theory.
- Applied knowledge and understanding: Students will be expected to use tensor formalism to carry out simple calculations or manipulations in the context of relativistic kinematics, classical electrodynamics, and classical field theory.
- Independent judgment: Students will develop critical thinking skills and the ability to choose the most appropriate tools, among those provided during the course, for solving specific problems.
- Communication skills: Students will be expected to acquire a correct and appropriate scientific language related to the topics covered in the course.
- Learning skills: Students will be able to deepen their understanding of specific concepts not explicitly covered during the course and to pursue advanced study independently using specialized scientific texts.

In detail, at the end of the course, the student

1. will be able to use the mathematical apparatus underlying the "covariant" formulations of special relativity (4-vectors, tensors), both for the theoretical discussion of fundamental concepts and for the resolution of kinematics and electromagnetism problems
2. will be able to connect the non-covariant formulation of electrodynamics with the covariant one

3. will be able to derive the equations of motion for a particle or for a field (scalar or vector) starting from the action and discuss the content and conservation laws of the energy-momentum tensor

Contents

Einstein's special relativity. Covariant formulation of the relativistic dynamics and of the classical electrodynamics. Relativistically-invariant Lagrangian formalism. Classical field theory: scalar and vector fields.

Detailed program

Introduction to Lorentz transformations. Relativistic kinematics. Covariant formulation of Special Relativity (4-vectors, tensors). Lorentz group.
Refs: Barone, Jackson (Weinberg).

- Quick recap of Classical Mechanics and Electromagnetism (EM) (principle of relativity, Galilean transformations, Maxwell's equations, wave equations). Non-invariance of EM under Galilean transformations. Quick discussion about ether hypothesis and role of the Michelson-Morley experiment.
- Bases of Special Relativity: intertial systems, synchronising clocks, postulates, events and intervals, invariance of the speed of light and Lorentz transformations.
- Recap on the main consequences of Lorentz transformations: time dilation, length contractions, proper time. Minkowski diagrams. Simultaneity, causality. Composition of velocities. Boosts in a generic direction.
- Brief discussion of the more famous "paradoxes" and of physical applications.
- Compact notation for Physics in Euclidean space: vectors, differential operators, various identities. Maxwell's equations (for fields and potentials) in compact notation.
- Special relativity in covariant notation: Minkowski space-time, metric, tensor calculus (covariant and controvariant vectors, tensors, the metric tensor, scalar quantities, differential operators). Basic concepts of differential geometry (+).
- Covariance ("invariance in form") of physical laws and the principle of relativity.
- Lorentz group: general properties, subgroups and classification of homogeneous Lorentz transformations. Generators and algebra of the restricted Lorentz group.
- Relativistic kinematics in covariant notation: 4-velocity, 4-acceleration, energy-momentum 4-vector and its properties. Einstein's relation between energy and mass, 4-momentum conservation.
- Relativistic kinematics: exercises and applications.
- Lorentz boosts in different directions and Thomas precession.

Relativistic dynamics of a particle; Maxwell's equations in covariant form.
Refs: Barone, Jackson (Weinberg, Landau)

- Dynamics of a relativistic particle: 4-force and force-acceleration equation.
- Maxwell's equation in covariant form: 4-current, continuity equation, 4-potential, gauge transformations, F?? tensor. Transformation laws of electric and magnetic fields. Invariants of the electromagnetic field.
- Covariant form of the Lorentz force. Interaction of EM fields with charged particles: motion in constant and uniform E and B fields.
- Charged particle with spin in an electromagnetic field.
- "Spin-orbit" interaction of an electron in a central field.
- Bargmann-Michel-Telegdi equation (+).
- Solution of the wave equation in covariant form (+).
- Radiation by moving charges (+).

Lagrangian formulation of the electrodynamics. Scalar and vector fields. Stress-energy tensor.
Refs: Barone, Jackson (Landau).

- Principle of stationary action and lagrangian formulation of the relativistic equations of motion for a free particle and for a charge in an electromagnetic field.
- Classical field theory: introduction and Euler-Lagrange equations.
- Scalar fields and Klein-Gordon equation.
- Stress-energy tensor.
- Vector fields: the Lagrangian of the electromagnetic field (free or interacting).
- The stress-energy tensor for the free and the interacting electromagnetic field.
- Noether's theorem (+).

(+) = advanced topic (covered only time permitting, the plan, though, is to cover all of them)

Prerequisites

Classical mechanics, classical electrodynamics, calculus.
(i.e. the content of the following courses: Fisica 1, Fisica 2, Analisi 1 and 2, Meccanica Classica, Matematica per la Fisica)

From a purely mathematical perspective, a strong background in the following topics is essential:

- basic linear algebra operations (e.g. determinant and inverse of matrix products, component representation of the cross product);
- first-order Taylor expansions;
- solving linear differential equations and evaluating elementary integrals;
- the Dirac delta function and its main properties.

Teaching form

Frontal teaching (lessons at the blackboard)

Textbook and teaching resource

Main textbooks:
Relativita'. Principi e Applicazioni, V. Barone

Classical Electrodynamics, J.D. Jackson
Chapter 11: Special Theory of Relativity
Chapter 12: Dynamics of Relativistic Particles and Electromagnetic Fields

Other useful textbooks:
Gravitation and Cosmology, S. Weinberg
Chapter 2: Special Relativity

The Classical Theory of Fields (Volume 2), L.D. Landau e E.M. Lifshitz

Chapter 1 to 4

Spacetime Physics, E.F. Taylor e J.A. Wheeler
Relevant parts available at the teacher's webpage.

- Various notes, complementary material, and exam sheets from previous years are available at the webpage https://virgilio.mib.infn.it/~re

Semester

First term.

Assessment method

The exam will consist of an oral test, with questions asked by the lecturer (it will not be possible to begin the oral exam by choosing a topic of your preference).

The oral exam will assess not only knowledge of all topics covered during the course, but also the ability to use appropriate formalism and to demonstrate a true understanding of the material. Students are expected to be able to apply the formalism to simple examples that are analogous and very similar, though not identical, to those discussed in class.

During the academic year, 5 exam sessions will be scheduled, in January, February, June, July, September.
A sixth session is organised, at an appropriate time, to facilitate students taking the third year exams.

The assessment method described above will apply by default to all students. Students who attended the course up to and including the 2024–2025 academic year may choose instead to take a written examination followed by an oral examination. To do so, they must inform the lecturer a few days before the exam date. They will then be notified in due course of the room where the written examination will take place. In the absence of explicit communication from such students, it will be assumed that they intend to take the new examination format (oral examination only).

If needed, Erasmus students are allowed to take the oral exam in English.

Office hours

By appointment.

Sustainable Development Goals

QUALITY EDUCATION