UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Geometry III

2627-3-E3501Q055

Aims

This course aims to yield an introduction to the fundamental group and differentiable manifolds.

Expected learning outcomes include:

(1) knowledge and comprehension: knowledge of definitions and basic results about the fundamental group and differentiable manifolds, and the knowledge of relevant examples. Comprehension of the main proof techniques.
(2) applying knowledge: ability to solve problems and discuss the geometric properties of concrete examples.
(3) autonomy: ability to find mistakes in proofs, ability to connect the content of the course with others subjects.
(4) communication: ability to express clearly and rigorously notions and definitions.
(5) learning skills: understading the correct approach to study the subject in order to apply the knowledge of the course to future problems.

Contents

Fundamental group and covering spaces of a topological space. Differentiable manifolds: topological manifolds, differentiable structures, tangent space, tangent bundle, calculus on manifolds. Immersions, submanifolds, critical and regular values. Vector fields, flows, Lie groups and Lie bracket. Differential forms, Stokes' theorem, De Rham cohomology.

Detailed program

The course is divided into 3 modules of 16 hours.

I: ***Fundamental group and coverings***

Definition of the fundamental groupoid and of the (based) fundamental group. Induced homomorphism by a continuous function. Basepoint dependence. Homotopically equivalent spaces and invariance of the fundamental group. Contractible spaces. 1-connected spaces. Coverings. Lifting theorem for continuous functions. Lifting of paths and homotopies. Fundamental group of the circle. Equivalence of coverings. Correspondence between coverings and subgroups of the fundamental group. Universal cover. Seifert Van-Kampen Theorem and presentation of groups. Properly discontinuous actions.

II: ***Smooth manifolds and vector fields***
Definition of topological manifold and smooth manifold. Smooth functions. Tangent space to a smooth manifold. Derivation at a point and curves. The differential of a smooth map. Regular and critical values of a smooth map. Immersions and embeddings. Submanifolds of a smooth manifold. Regular value theorem and constant rank theorem. Lie groups and subgroups. Tangent bundle to a smooth manifold. Vector fields and derivations. Lie bracket. Completeness of a vector field. Integral curve and Cauchy type theorem. Flow of a vector field. Invariant vector fields on a Lie group. The Lie algebra of a Lie group.

III: ***Differential forms***
Cotangent bundle and exterior powers. Differential forms. De Rham cohomology: closed and exact forms. Orientation on a smooth manifold. Integration of compactly supported differential forms. Partition of unity. Stokes' theorem. De Rham cohomology in top degree. The degree of a smooth map. The degree formula. Invariance of De Rham cohomology and Poincaré lemma. Exact sequences of vector spaces. The exact sequence of Mayer Vietoris.

Prerequisites

Fundamental prerequisite is the content of the courses of Geometry I and II, Mathematical Analysis I and II, Linear Algebra and Geometry, Algebra I.

Teaching form

Blackboard lectures (6 ECTS, 48 hours), in class. In italian.

Textbook and teaching resource

Textbooks

For the first module:

1. G. Bredon - *Topology and Geometry (Chapter 3)*
2. J. Munkres - *Topology (Chapter 9-11-13)*
3. A. Hatcher - *Introduction to algebraic topology (Chapter 3)*

For the second and the third modules:

1. L. Tu - An introduction to Manifolds.

2. J. Lee - Introduction to Smooth Manifolds.
3. R. Bott, L.Tu - Differential forms in algebraic topology. (Chapter 1).

There are also the notes of the course.

Semester

First semester.

Assessment method

Written examination: 4 exercises with a maximum of 28 points. You need 14 points to do the oral examination. The exercises are typically focused on computing something, give some definitions and examples, write small proofs (from the course).
Oral examination: 2 questions about the main topics of the course or about the exercises of the written exam. The exam aims to verify the knowledge and the quality of the exposition of the candidate.
**Both examinations are compulsory. **
There are no mid-term exams.

Office hours

By appointment, to be organized with the teacher.

Sustainable Development Goals

QUALITY EDUCATION