UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematical Physics

2627-3-E3501Q059

Aims

The course aims at showing how physical phenomena of paramount importance are described through partial differential equations, at teaching the methods for finding solutions and the study of the properties of such solutions.

In terms of Dublin descriptors, the aims are:

1 (Knowledge and understanding) The student will gain knowledge of the fundamental properties of the solutions of the wave equation, the heat equation and Laplace equation, as well as the notion of distribution.

2 (Applying knowledge and understanding) The student will be able to apply the methods studied to the dolution of problems of interest for mathematical physics and related, simple applications. He will acquire familiarity with the techniques of solution and with the different qualitative behaviors of the solutions.

3 (Making judgements) Attention is paid, in particular, to be sure that the student learns to read the qualitative properties of the solutoons of the proposed problems and is able to understand on his own whether they provide a plausible representation for the described phenomenon.

4 (Communication skills) The student must be able to present the fundamental concepts of the course clearly and rigorously, using correct mathematical language.

5 (Learning skills) The student will become able to pursue further studies on his own, making use of suitable scientific text, in order to tackle different physical phenomena through partial differential equations, with the degree of rigor, typical of mathematical physics.

Contents

Introduction to classical partial differential equations of mathematical physics and to the related models: wave equation, heat equation, Laplace equation. Weak solutions and distributions.

Detailed program

- Introduction to partial differential equations: Maxwell equations, continuity equation and Euler equation.
- Continuity equation: initial value problem solution and method of characteristics.
- Wave equation: deduction from the model of vibrating string and of chain of harmonic oscillators, 1-dimensional solutions, characteristics and causal cone, Lorentz invariance, effects of sources and boundary conditions, well posedness, dependence on the dimension of the space, Huygens principle and Kirchhoff solution, Liénard-Wiecher potentials.
- Heat equation: physical meaning, self-similar solutions, fundamental solutions and initial value problem solution, weak maximum principles, effects of sources and boundary conditions, well posedness.
- Solutions in bounded intervals of heat and wave equations, symmetric operators and dependence on boundary conditions.
- Comparison between heat and wave equations, dispersion relations.
- Laplace equation: radial solutions, Green identities, properties of harmonic functions, Dirichlet principle, boundary conditions and compatibility conditions.
- Poisson equation: representation formula and general solution, Green functions, method of image charges.
- Distributions: definition and fundamental properties, Dirac delta and Green functions, weak solutions, tempered distributions and Fourier transform.

Prerequisites

Elements of classical Analysis (I & II). Elements of finite dimensional Euclidean geometry. Elements of Physics (I & II)-

Teaching form

48 hours (6 ETCS) of lecture-based, in-person, teaching. The theoretical part is integrated by examples and exercises. Lectures will be delivered in italian.

Textbook and teaching resource

-Textbook:

W. Strauss Partial differential equations, Wiley&Sons

Some lecture notes will be also provided. Further suggested readings:

S.Salsa, Partial differential equations in action, Springer

L.C. Evans, Partial differential equations, AMS

G. B. Whitham, Linear and nonlinear waves, Wiley&Sons

Semester

Second

Assessment method

The exam is individual and is divided in a written and an oral part, both of them are required. In the written exam the proficiency in solving exercises and problems similar to those discussed in the lectures is evaluated. The oral exam is focused on assessing the understanding of the mathematical concepts and their derivation, by asking the statements and the proofs of theorems, relevant examples and deductions of equations from physical examples.

Office hours

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Sustainable Development Goals

QUALITY EDUCATION