UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

# COURSE SYLLABUS

## Numerical Analysis

**2627-3-E3501Q058**

### Aims

In line with the educational objectives of the Bachelor Degree in Mathematics, the course aims at providing the basic knowledge, with a deep theoretical support, about the topics of the course (mainly optimization problems, and also discretization of ordinary differential equations). It will also build the skills needed to understand, analyse and compare the different methods, in addition to implementing them in the computer.

Learning Objectives according to the Dublin Descriptors

1. Knowledge and understanding
   Students will acquire a basic but solid understanding of numerical analysis tools, particularly in optimization and the numerical treatment of ordinary differential equations. The course offers both theoretical foundations and practical applications, with a focus on understanding the behavior and limitations of numerical methods.

2. Applying knowledge and understanding
   Students will be able to apply the studied methods to solve basic optimization problems (with and without constraints) and to discretize and solve ordinary differential equations. They will also be able to implement algorithms in a computing environment (such as MATLAB) and test their performance on practical examples.

3. Making judgements
   The course develops the ability to critically evaluate the numerical methods introduced, selecting the most suitable ones for specific problems. Students will learn to interpret numerical results with a critical approach, especially with respect to approximation errors and parameter choices.

4. Communication skills
   Students will learn to clearly describe the numerical techniques used and the results obtained, using appropriate mathematical and technical language. They will be able to orally present the outcomes of their work, including activities performed in the lab.

5. Learning skills
   By the end of the course, students will have developed the skills needed to independently approach more advanced numerical methods and apply the acquired knowledge to further studies.

## Contents

The main part of the course is about optimization problems in R?, whose resolution is a fundamental step in many applied math problems. We will consider the following topics: search for zeros of functions, then minima of functions, finally constrained minima. The last part of the course will instead consider the discretization of ordinary differential equations.

The course will provide a rigorous theoretical support of the methods considered, together with a computational lab part in MATLAB.

## Detailed program

All the topics developed in class will have also a coding part in the computer Lab (MATLAB language). Some labs will consider PDE problems that, after discretization/approximation by some numerical scheme, become optimization problems in R^N. We will consider the following topics. Iterative methods for fixed points, local and global convergence properties. Search of zeros of vector valued functions, quasi-Newton methods, examples, local convergence, modifications for global convergence. Search of minima of functions (in open sets), line search methods, examples, convergence properties. Search of constrained minima, Kuhn-Tucker and lagrangian theory, projected gradient, Uzawa method, convergence properties. Ordinary differential equations, one step methods, convergence theory, absolute stability, RK methods.

## Prerequisites

The standard knowledge of a third year math student is sufficient

## Teaching form

Standard blackboard classes, plus coding lessons with projector.

## Textbook and teaching resource

• C.T. Kelley, “Iterative methods for linear and nonlinear equations”, SIAM

• J. Nocedal, S.J. Wright, “Numerical Optimization”, Springer

• P.G. Ciarlet, “Introduction to numerical linear algebra and optimizations”, Cambridge Texts in Applied Math

• Uploaded pdf text on the Ordinary Diff. Eq. part

## Semester

Second semester.

## Assessment method

The exam is an oral examination, and is divided into two parts. In the first part, the student presents a matlab laboratory project (to be developed individually), chosen by the teacher among a set of three previously selected by the student (these are 3 among the projects developed in the Lab during the course). The second part is an evaluation of the critical and operational knowledge of the definitions, results and proofs presented during the course. The relative weight of the project and the theoretical examination are roughly 30% and 70%, respectively. In the project discussion the teacher will evaluate the exactness of the results and the comprehension of the practical/computational aspects of the adopted numerical method. During the theoretical part of the exam, the teacher will mainly evaluate the comprehension of the topic and the mathematical rigour in presenting the numerical methods and the associated proofs.

There will not be any mid-course evaluation/exam during the course.

## Office hours

Flexible, arranged directly via email.

## Sustainable Development Goals