UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Mathematical Statistics

2627-3-E3501Q062

Aims

Knowledge and understanding
The student will acquire a solid understanding of the main concepts of inferential statistics, with particular focus on estimator theory, Fisher information, confidence intervals, and hypothesis testing. The course will deepen the formal framework of statistical models and the theoretical foundations of estimation and inference, promoting a rigorous mathematical comprehension of statistical methods.

Applying knowledge and understanding
The student will be able to apply statistical techniques critically to real-world problems, selecting appropriate models and using computational tools (especially the R software) for data analysis. They will also be able to evaluate the efficiency and properties of estimators within specific models.

Making judgements
The student will develop the ability to autonomously analyze inferential problems, critically assessing model assumptions and the implications of methodological choices. They will be able to compare different inferential methods and justify their decisions in both theoretical and applied contexts.

Communication skills
The student will be able to clearly and rigorously present theoretical and applied topics in mathematical statistics, using appropriate technical language. They will be able to write reports on statistical data analysis and effectively communicate results to both specialists and non-specialists.

Learning skills
The student will develop the skills necessary for independent study of advanced texts in statistics and for deepening theoretical and applied aspects of the field, also in view of pursuing graduate studies or engaging in research activities.

Contents

The course begins with brief reviews of probability theory covering random variables, main distributions, moments, and Gaussian vectors. The core syllabus then focuses on mathematical statistics: sampling theory and sufficient statistics; point estimation through consistency, mean squared error, and the method of moments; and interval estimation via confidence intervals. Finally, the theory of hypothesis testing is developed, including both exact tests based on the likelihood ratio and asymptotic tests for large samples using the properties of maximum likelihood estimators, such as the Wald and asymptotic LRT tests.

Detailed program

Review of Probability
? Discrete and absolutely continuous random variables, with their respective densities.
? Detailed analysis of notable discrete distributions: Bernoulli, Binomial, Geometric, Poisson, and Discrete Uniform.
? Analysis of notable continuous distributions: Uniform, Exponential, Gamma, and Normal (Gaussian).
? Calculation of n-th order moments, expected value, variance, and introduction to characteristic functions.
? Study of Gaussian vectors: definition via linear combinations, mean vector, covariance matrix, and properties of stochastic independence related to uncorrelation.

Sampling Theory and Sufficient Statistics
? Definition of a statistical sample extracted from a population and formalization of independent and identically distributed (i.i.d.) random variables.
? Probabilistic structure of the sample: joint density.
? Study of fundamental sample statistics: sample mean and sample variance, with analytical verification of their properties and respective expected values.
? Formal definition of a sufficient statistic for an unknown parameter and application of the factorization criterion for identifying sufficiency.
Point Estimation Theory
? Definition of a point estimator and systematic study of the property of consistency.
? Operational reviews of the Weak Law of Large Numbers (WLLN) and the main concepts of convergence (in probability and in distribution).
? Mean Squared Error (MSE) as a measure of accuracy: decomposition into variance and bias of the estimator.
? Optimality criteria in estimation: search for Minimum Variance Unbiased Estimators (MVUE / BUE).
? Classical methodologies for the construction of point estimators: in-depth study of the method of moments.

Interval Estimation (Confidence Intervals)
? Framing the interval estimation problem: definition of a pair of statistics to delimit the parameter of interest.
? Formal definition of the confidence level and the minimal confidence coefficient.
? Constructive methods for exact confidence intervals and application examples on notable models (e.g., estimation of frequency in Bernoulli models or parameters of Gamma models).

Statistical Hypothesis Testing (Exact Theory)
? Logical-mathematical formulation of statistical hypotheses: partition of the parameter space into null hypothesis (H0) and alternative hypothesis (H1).
? Definition of the rejection region and formalization of decision errors: Type I Error (rejecting H0 when it is true) and Type II Error (accepting H0 when it is false).
? Definition and study of the power function of the statistical test and size of the test.
? Structure of the Likelihood Ratio Test (LRT) in exact contexts and the role of sufficient statistics in the reduction of the sample space.

Asymptotic Theory and Large Samples
? Analysis of the behavior of families of estimators as the sample size tends to infinity: properties of consistency

and asymptotic normality.
? Maximum Likelihood Theory: definition of the log-likelihood function and construction of Maximum Likelihood Estimators (MLE).
? Study of Fisher Information (expected and observed) and its link to the asymptotic efficiency of maximum likelihood estimators.
? Properties of local and global consistency of the maximum point of the log-likelihood under appropriate regularity conditions.

Asymptotic Tests
? Construction of statistical tests based on approximations for large samples.
? Asymptotic behavior of the likelihood ratio statistic: the theorem on the convergence of the statistic -2 log(?) to a Chi-squared distribution under the null hypothesis.
? The Wald Test based on the asymptotic normality of MLE estimators.

Introduction to R

Prerequisites

Mathematical Analysis I and II, with a focus on integral calculus.

Basic Probability: laws of discrete and continuous random variables. Expected value and variance. Law of functions of random variables. Independence. Convergence of sequences of random variables.

Teaching form

A hybrid teaching approach is used, that combines lecture-based teaching (DE) and interactive teaching (DI). DE (80%) involves detailed presentation and explanation of theoretical content. DI includes active student participation through exercises and problems, short presentations, group discussions, and group or individual work.
The lessons (48 hours, 6 ECTS) are conducted in person and are held in Italian. Upon request and by mutual agreement between the instructor and students, it will be possible to use the English language.
Depending on the availability of recording equipment, the lessons will be recorded and made available on the e-learning website.

Textbook and teaching resource

Lecture notes

Statistical inference / George Casella, Roger L. Berger. https://www.google.com/url?sa=t&source=web&rct=j&opi=89978449&url=https://pages.stat.wisc.edu/~shao/stat610/Casella_Berger_Statistical_Inference.pdf&ved=2ahUKEwjJjq7Bk4yVAxUp1wIHHQmmCQYQFnoECA4QAQ&usg=AOvVaw3vA7hSUPxpugJ-ZDtidArd

Introduction to Statistics by A.M. Mood, F.A. Graybill, D.C. Boes, 1991, McGraw-Hill Italia, ISBN: 9788838606618

An Introduction to Mathematical Statistics: F. Bijma, M. Jonker, A. van der Vaart, Amsterdam University Press

Other materials: Lecture slides and exercise materials on elearning.unimib.it

Semester

Second semester

Assessment method

Exam Structure and Rules
The exam consists of two parts: a mandatory written exam and an optional oral exam.

The Written Exam
Requirement: It is mandatory. You must score 16 or higher to pass.

Format: The written test consists of:

Open-ended questions: Focusing on the theoretical concepts covered throughout the course, including stating definitions and/or stating and proving theorems.

Written exercises: Applying theoretical concepts and techniques similar to the exercises covered in class and assigned as homework.

Allowed Materials: During the written exam, you are not permitted to use teaching materials, with the exception of a specific formula sheet provided on Moodle.

The Oral Exam
It is mandatory: * If you wish to achieve a final grade higher than 28 and if you scored lower than 20 on the written exam.

General rule: Both the professor and the student reserve the right to request an oral exam, regardless of the written test score. The oral exam must take place within the same examination session.

Content: It is a discussion based on your written exam, the topics covered during lectures, and the course report.

Evaluation: The oral exam assesses the exact same qualities in your answers as those evaluated in the written exam.

Office hours

By appointment

Sustainable Development Goals

QUALITY EDUCATION