UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Algebra III

2627-3-E3501Q054

Aims

(1) Knowledge and capabilities of comprehension: The students will acquire a solid knowledge of the main concepts, results and applications in Galois theory. Special interest concerns the study of algebraic field extensions, elementary finite group theory, the existence and uniqueness of the algebraic closure of a field, properties like normality and separability, the Main Theorem in Galois theory and an intensive discussion of cyclotomic field extensions. This will result in several remarkable applications resolving notable problems in classical elementary geometry. Theoretical knowledge acquisition will proceed with mathematical rigor and will be supported by concrete examples, applications, questions and open problems. For the convenience of the student the lecture will also include a short discussion of the basic “axiomatic” of modern mathematics (Zorn’s lemma and ZFC) supported by basic examples and concrete applications.

(2) Capabiliities of applying knowledge and comprehension: The student will be able to apply the principle notions and results in Galois theory to concrete mathematical problems. The student will be offered activities in order to improve algebraic and elementary geometric techniques.

(3) Independence of judgement: The lecture intends to stimulate and develop the capability of the student to analyze critically mathematical assertions and proofs. Special emphasis is laid on the capability to discover the situation in which a special result can be applied and to explain the motivation to introduce certain notions and concepts.

(4) Communication skills: The student will be able to explain and discuss orally the principle notions and results in Galois theory in a coherent and rigorous manner using the scientific mathematical language. This will also include proofs of the main results in Galois theory as well as the principle applications.

(5) Learning capabilities: The lecture will offer the student the means for proceeding the study of Algebra on a higher level facing new arguments and problems sistematically and with rigor making use of the progressing knowledge. The usual resources (notes, handouts and books ..) will be used for increasing and developing the student’s competences.

Contents

The theory of algebraic field extensions and Galois theory

Detailed program

The algebraic closure of a field,
normal and separable extensions of fields,
the existence of primitive elements in finite separable field extensions,

The fundamental theorem in Galois theory,
cyclotomic extensions,
Finite and solvable groups,
polynomial equations of degree at least 5 are in general
not solvable,
the field of elements in C constructable by compass and ruler,
the field of complex numbers is algebraically closed.

Prerequisites

Prerequisites: The contents of the courses *Linear algebra and Geometry*, *Algebra I* and *Algebra II*.

Teaching form

The lectures will be delivered in presense in a lecture hall in Italian.
6 credits (ECTS) of lecturing.
In hours: 48 hours of lecturing.

Textbook and teaching resource

N. Jacobson, Basic Algebra I, Freeman & Co, 1985.

Additional References:

S. Bosch, Algebra, Springer-Verlag, 2003.

Semester

1?? semester

Assessment method

Examination: **oral examination** of ca. 20 minutes on the content of the course.

The questions will concern definitions, examples, counterexamples, exposition and application of Theorems as well as their proofs.

Office hours

On appointment

Sustainable Development Goals

QUALITY EDUCATION