UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

SYLLABUS DEL CORSO

Mathematical Methods for Applied Physics

2627-2-E3004Q008

Aims

Knowledge and understanding: The student will learn fundamental mathematical concepts and advanced calculus techniques. The student will become familiar with some of the main mathematical tools for the study of physical systems: differential and integral calculus in the complex plane, operators in infinite dimensional spaces useful for the formulation of Quantum Mechanics, and some of the most common integral transforms, like the Fourier and Laplace transforms.
Applying knowledge and understanding: The student will learn to apply the acquired mathematical tools to the study of physical systems and their technological applications. The student will learn how to handle advanced programs of analytical calculus.
Making judgments: The student will develop critical thinking and judgment skills in selecting the most appropriate tool, among those provided during the course, to solve a specific problem.
Communication skills: The student will be expected to acquire a correct and appropriate scientific language suited to the topics covered in the course.
Learning skills: The student will be able to deepen their understanding of specific concepts not covered during the course and to independently pursue advanced study using specialised scientific texts.

Contents

Complex Analysis
Holomorphic functions
Contour integration
Cauchy's theorem
Laurent series and residues
Analytic continuation
Applications to evaluating integrals

Functional Analysis and Transforms
Hilbert spaces and Lp spaces
Linear, self-adjoint, and unitary operators
Spectral theory
Fourier series and Fourier transforms
Laplace transforms

Detailed program

Complex Analysis
The complex plane
Maps in C
Complex-valued functions of a complex variable
Complex differentiability
Cauchy–Riemann conditions
Holomorphic functions on an open subset of C
Integration in the complex plane
Cauchy's theorems
Laurent series expansions
Zeros and isolated singularities
Behavior of functions near isolated singularities
Poles
Casorati–Weierstrass theorem for essential singularities
Liouville's theorem
Definition of residue
Residue at infinity
Calculation of residues
Residue theorem
Techniques for evaluating real integrals through analytic continuation into C
Jordan's lemma
Analytic continuation and multivalued functions

Functional Spaces
Review of topological spaces, metric spaces, and Banach spaces
Sequence spaces lp and l?
Hilbert spaces
Inner products and orthogonality
Complete orthonormal systems
Fischer–Riesz theorem
Lp and L?
Important orthonormal systems:
Fourier series
Hermite polynomials
Legendre polynomials
Laguerre polynomials
Bessel functions

Linear Operators
Linear operators on Hilbert spaces and their properties
Continuous and bounded operators
Operator norm
Spectral problems and classification of eigenvalues

Definition of eigenfunctions
Self-adjoint operators
Eigenvalues and eigenfunctions of self-adjoint operators
Spectral decomposition theorem
Projectors and their properties
Isometric operators and their properties
Unitary operators and their properties
Linear functionals on Hilbert spaces
Dual space
Riesz representation theorem

Fourier Transform
Definition on L1 and L2 and main properties
Techniques for computing Fourier transforms
Applications to solving differential equations relevant to physics

Laplace Transform
Definition and main properties
Inverse transform
Techniques for computing Laplace transforms
Applications to solving differential equations relevant to physics

Prerequisites

Knowledge of the basic contents of the first year mathematics courses

Teaching form

Online synchronous and asynchronous lectures, supported by in-person tutorial sessions in Pavia.

Textbook and teaching resource

Lecture notes and PDF files from lectures and exercise sessions are available on the course e-learning page.
Main References
G. Cicogna, Mathematical Methods of Physics, Springer
M. R. Spiegel, Complex Variables, Schaum Outline Series
J. Bak & D. J. Newman, Complex Analysis, Springer
L. Debnath & P. Mikusi?ski, Hilbert Spaces with Applications, Elsevier
M. R. Spiegel, Fourier Analysis, Schaum Outline Series
G. Pradisi, Lectures on Mathematical Methods of Physics, Edizioni della Normale

Semester

Second semester

Assessment method

The exam consists of:
A comprehensive written exam covering the entire syllabus (5 exercises in 3 hours)
A mandatory oral examination
Students may take the oral exam only if they obtain a written exam grade of 15/30 or higher.
The examination process must be completed within six months from the written exam date.

Office hours

By appointment, via email: silvia.penati@unimib.it, or silvia.penati@mib.infn.it

Sustainable Development Goals

QUALITY EDUCATION