UNIVERSITÀ
DEGLI STUDI DI MILANO-BICOCCA

COURSE SYLLABUS

Computational Physics

2627-2-E3004Q010-E3004Q01001

Aims

This course aims to bridge the gap between theoretical physics and numerical experiments by treating the computer as a strict physics laboratory. Students will learn to confidently implement numerical methods to simulate complex physical systems, with a heavy emphasis on rigorously quantifying computational errors. The objectives according to Dublin Descriptors are:

- **Knowledge and understanding**: Understand the mathematical limits of discrete computation, including truncation versus round-off errors, the necessity of symplectic integrators, and the Nyquist-Shannon limits in spectral analysis.
- **Applying knowledge and understanding:** Translate physical laws (ordinary differential equations, partial differential equations, stochastic processes) into working numerical code to simulate multi-dimensional and non-linear physical systems.
- **Making judgements**: Develop the critical ability to distinguish between genuine physical phenomena and numerical artifacts (e.g., algorithmic energy generation, grid aliasing) through quantitative and qualitative error analysis.
- **Communication skills**: Communicate scientific results clearly by compiling Jupyter Notebooks that logically combine executable code, mathematical hypotheses, and graphical data representation.
- **Learning skills**: Develop autonomous debugging and algorithmic problem-solving skills, allowing students to independently assess the validity of computational models in future research setups.

Contents

- **The Limits of Discretization**: Algorithmic limits and symplectic geometry.
- **The Unpredictable Universe**: Non-linear dynamics and deterministic chaos.
- **Statistical Mechanics**: Random walks, diffusion, and statistical noise.
- **Invisible Fields**: Grid discretization and finite difference methods for PDEs.

- **Orbital Dynamics**: The Restricted 3-Body Problem and non-linear error amplification.
- **Spectral Analysis**: Discrete Fourier Transforms and normal modes extraction.

Detailed program

- **Part 1: The Limits of Discretization (Kinematics & ODEs)** Theory: The anatomy of floating-point numbers. Truncation error vs. round-off error. Taylor series expansions for algorithms (Euler, Runge-Kutta 4, Verlet). The concept of symplectic integrators. Lab experiment: Simulating a simple harmonic oscillator. Plotting Total Energy over time to discover artificial "numerical friction". Deriving scaling laws via log-log plots of global error versus time-step size.
- **Part 2: The Unpredictable Universe (Chaos & Non-Linear Dynamics)** Theory: Non-linear differential equations, phase space trajectories, Poincaré sections, and an introduction to Lyapunov exponents. Lab Experiment: The Driven Damped Pendulum. Mapping the transition from periodic motion to deterministic chaos and generating bifurcation diagrams. Measuring sensitivity to initial conditions.
- **Part 3: The Apollo Problem (Orbital Dynamics)** Theory: Newton's Law of Universal Gravitation, Kepler's orbits, and the transition to the Restricted 3-Body Problem. Numerical Experiment: Simulating the Earth-Moon system and finding the exact launch velocity and angle for a lunar slingshot. Analyzing the amplification of floating-point precision errors in non-linear orbital trajectories.
- **Part 4: From Micro to Macro (Statistical Mechanics)** Theory: Pseudo-random number generation, random walks, and the Central Limit Theorem. Lab Experiment: Simulating independent random walkers in 2D to calculate mean squared displacement. Verifying statistical noise scales proportionally to 1/N.
- **Part 5: Invisible Fields (Electrostatics & PDEs)** Theory: Discretizing space. Finite difference methods for Elliptic Partial Differential Equations. Dirichlet vs. Neumann boundary conditions. Relaxation algorithms (Jacobi and Gauss-Seidel). Lab Experiment: Solving Laplace's equation for a realistic parallel-plate capacitor on a numerical grid. Calculating grid resolution limits necessary to achieve convergence with analytical approximations.
- **Part 6: Listening to Physics (Spectral Analysis & FFT)** Theory: The Discrete Fourier Transform (DFT), Fast Fourier Transform (FFT), Nyquist-Shannon sampling theorem, and spectral leakage. Lab Experiment: Generating long-term time-domain data for simple, damped, and coupled oscillators. Using the FFT to observe Lorentzian curves, extract higher-order odd harmonics in non-linear regimes, and extract normal modes from chaotic signals.

Prerequisites

Knowledge of basic Calculus, Classical Mechanics, and Thermodynamics is required. Furthermore, students must have successfully completed the first-year "Physical Modelling and Simulations" course, possessing a strong foundational knowledge of Python, NumPy, Matplotlib, and basic numerical integration methods.

Teaching form

The course is structured as 64 total hours, utilizing a mix of methodologies:

- **16 hours of Asynchronous Remote Lectures**: Recorded video lectures focusing on theoretical mathematical derivations and algorithm formulation.
- **48 hours of In-Person Laboratory Sessions**: Hands-on coding sessions in the computational lab. Each 8-hour module is split into two phases: "Apparatus Building" (programming the algorithms) and "Numerical

Experiments" (extracting physical measurements and performing error analysis).

Textbook and teaching resource

- Lecture notes, asynchronous videos, and Jupyter Notebook templates provided in class.
- Computational Physics: Problem Solving with Python by Landau, Páez & Bordeianu (2024)
- Computational Physics by Newman (2013 or 2025).
- Computational Physics 2nd Edition by Giordano and Nakanishi (2006)
- Numerical Recipes by Press et al. (2007)

Semester

Second semester

Assessment method

The assessment evaluates the student's ability to implement computational models and critically assess their physical validity. The exam is structured as follows:

- **Mandatory Lab Portfolio**: Throughout the course, students must submit a computational lab report (Jupyter Notebook) for each module. Complete submission of this portfolio is a strict prerequisite to access the final exam, ensuring continuous engagement with the material.

- **Final Oral Exam**: A final individual oral examination divided into two main phases:

    - **Project Defense**: Presentation and discussion of a final numerical project chosen by the student (either an advanced extension of a course module or the simulation of a new physical system).

    - **Portfolio Review**: The commission will randomly select one or more notebooks from the student’s submitted lab portfolio. The student will be asked to navigate their own code, justify their algorithmic choices, explain specific blocks of logic, and interpret the physical meaning of their error analysis plots.

- **Grading Criteria**: The final grade is determined by the logical architecture and successful execution of the algorithms across both the final project and the random portfolio review (30%), the accuracy of the physical simulations (20%), the depth and rigor of the numerical error analysis (30%), and the clarity and critical thinking demonstrated during the oral discussion covering the material studied in the course (20%).

Office hours

By appointment via email.

Sustainable Development Goals

QUALITY EDUCATION **|** INDUSTRY, INNOVATION AND INFRASTRUCTURE